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ON THE DISTRIBUTION OF THE NUMBER OF LATTICE POINTS INSIDE A FAMILY OF CONVEX OVALS

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1. Introduction. Let γ be a simple convex closed smooth curve defined in the plane $x \in \mathbf{R}^2$ by an equation

$$x = X(t), \qquad 0 \leqslant t \leqslant 1 \qquad (X(0) = X(1)).$$

Let γ_R, $R > 0$, be the curve defined by the equation

$$x = RX(t);$$

i.e., γ_R is the homothety of γ with the coefficient R. Let $\Omega_\gamma(R)$ be the domain enclosed by γ_R, $\alpha \in \mathbf{R}^2$ be a fixed point in the plane, $\alpha + \mathbf{Z}^2 = \{x = \alpha + n, n \in \mathbf{Z}^2\}$ be a shifted square lattice, and

$$N_\gamma(R; \alpha) = |\Omega_\gamma(R) \cap (\alpha + \mathbf{Z}^2)|$$

be the number of lattice points lying within γ_R. Finally, let

$$F_\gamma(R; \alpha) = \frac{N_\gamma(R; \alpha) - \operatorname{Area} \Omega_\gamma(R)}{\sqrt{R}}. \tag{1.1}$$

We are interested in the distribution of $F_\gamma(R; \alpha)$ on the half-line $\{R > 0\}$. Our main result is the following theorem.

THEOREM 1.1. *Let γ be a simple C^7-smooth (i.e., $X(t) \in C^7([0, 1])$) closed convex curve with positive curvature, such that the origin lies inside γ, and let α be a fixed point in the plane. Then for every probability density $p(x)$ on $[0, 1]$ and every bounded continuous function $g(x)$ on $\mathbf{R}^1$,*

$$\lim_{T \to \infty} \frac{1}{T} \int_0^T g(F_\gamma(R; \alpha)) p(R/T)\, dR = \int g(x) \nu_\gamma(dx; \alpha) \tag{1.2}$$

where $\nu_\gamma(dx; \alpha)$ is a probability distribution on $\mathbf{R}^1$, which does not depend on $p(x)$ and $g(x)$. In addition,

$$\lim_{T \to \infty} \frac{1}{T} \int_0^T F_\gamma(R; \alpha) p(R/T)\, dR = \int x \nu_\gamma(dx; \alpha) = 0 \tag{1.3}$$

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