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COMPLEX HÉNON MAPPINGS IN $\mathbb{C}^2$ AND FATOU-BIEBERBACH DOMAINS

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Introduction. For $(a, c) \in \mathbb{C}^* \times \mathbb{C}$ the formula

$$g(z, w) = (z^2 + c + aw, z)$$

defines a biholomorphism in $\mathbb{C}^2$ whose Jacobian is $-a$. These are the complex continuations of the maps studied by Hénon when $(z, w) \in \mathbb{R}^2$ and $(a, c) \in \mathbb{R}^* \times \mathbb{R}$.

In [H] Hubbard introduced the following terminology. Let

$$K^{\pm} = \{p; p \in \mathbb{C}^2, g^{\pm n}(p) \text{ is a bounded sequence}\}.$$

Also let $J^{\pm} = \partial K^{\pm}$ and $K = K^+ \cap K^-$. As in [Br] and [H], we define the functions

$$G^+(z, w) = \lim_{n \to \infty} \frac{1}{2^n} \log^+ \|g^n(z, w)\|$$

and

$$G^-(z, w) = \lim_{n \to \infty} \frac{1}{2^n} \log^+ \|g^{-n}(z, w)\|.$$

It was shown in [H], [BS1], that $G^{\pm}$ are continuous functions in $\mathbb{C}^2$ plurisubharmonic in $U^+ = \mathbb{C}^2 \backslash K^+$ and $U^- = \mathbb{C}^2 \backslash K^-$, respectively. It follows that K^+ and K^- are nonpluripolar closed sets.

Define $\mu^{\pm} = dd^c G^{\pm}$. The positive, closed (1, 1) currents $\mu^{\pm}$ satisfy the functional equations

$$g^* \mu^{\pm} = 2^{\pm 1} \mu^{\pm}.$$

It was shown by Bedford and Smillie [BS2] that, if V is an algebraic curve in $\mathbb{C}^2$, then the currents $(1/2^n)[g^{-n}(V)]$ converge to a constant multiple of μ^+. Assuming that g is hyperbolic on $J = J^+ \cap J^-$, Bedford and Smillie showed that the interior of K^+ consists of the basins of finitely many sink orbits and that J^+ has a foliation $\mathscr{F}^+$ whose leaves are complex manifolds biholomorphically equivalent to $\mathbb{C}$.

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