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# SHARP ESTIMATES FOR SOME FUNCTIONS OF THE LAPLACIAN ON NONCOMPACT SYMMETRIC SPACES

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**0. Introduction.** In a series of papers ([Ta1], [CGT], [DST], [Ta2]) M. E. Taylor has developed a successful method of studying functions of the Laplacian on noncompact Riemannian manifolds. Let us recall that his approach is based on wave equation techniques and that it produces remarkable $L^1$ kernel estimates and some weaker pointwise results. We recently found the group theoretical counterpart of M. E. Taylor's arguments in the particular context of Riemannian symmetric spaces of the noncompact type $G/K$, and this led us, for instance, to a perfect analogue of the Hörmander-Michlin multiplier theorem for functions of all invariant differential operators on $G/K$ ([An3]).

In this paper we shall refine our technique in order to get sharp pointwise kernel estimates for some classical functions of the Laplacian $\Delta$ on $G/K$, like the heat semigroup $e^{t\Delta}$, the Poisson semigroup $e^{-t(-\Delta)^{1/2}}$, the Riesz transforms $\nabla^i \circ (-\Delta)^{-i/2}$, the resolvent $(zI - \Delta)^{-1}$, and, more generally, the potentials $(zI - \Delta)^{-s}$. Our estimates are by far the best ever obtained for general Riemannian symmetric spaces of the noncompact type. They should be close to optimal, as suggested by particular cases, where explicit expressions are available. In any case they are sharp enough to imply delicate endpoint results such as the weak $L^1 \to L^1$ inequality for the heat maximal operator, for the Littlewood-Paley-Stein $g$-function, or for the first two Riesz transforms.

Some words about our method. The main problem occurring while estimating our kernels $\mathscr{k}$ lies at infinity. As in [An3], this is dealt with by combining $L^2$ identities (the Plancherel formula in both the Euclidean and the non-Euclidean setting) with a support conservation property for the Abel transform. The difference lies in the cutoff on the Abel transform side, which is now more subtly defined and drags us actually into a lot of technicalities. What comes out eventually is a pointwise control of $\mathscr{k}$ in terms of $L^2$ conditions on its Abel transform $\mathscr{A}\mathscr{k}$. This leads to sharp kernel estimates, especially away from the walls, when $\mathscr{A}\mathscr{k}$ is exponentially decreasing at infinity, which is always the case for us.

We come now to the organization of our paper. In Section 1 we recall the basic notation and some general facts about the Fourier transform on $G/K$, notably our improved version of the bi-$K$-invariant Paley-Wiener theorem. In Section 2 we develop a method to get pointwise estimates at infinity for functions of the

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