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BOUNDARY MORERA THEOREMS FOR HOLOMORPHIC FUNCTIONS OF SEVERAL COMPLEX VARIABLES

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0. Introduction. This paper is devoted to the study of the boundary values of holomorphic functions of several complex variables. The subject is hardly new, but it continues to fascinate.

The particular issues we are concerned with here arise from reflection on some earlier work of the authors that concerns the one-dimensional extension property. If D is a domain in a complex manifold, we will denote by $A(D)$ the algebra of functions continuous on $\bar{D}$ and holomorphic on D. Given a bounded domain D in $\mathbb{C}^N$, a continuous function f on bD is said to enjoy the one-dimensional extension property with respect to a complex line Λ that meets D if there is a function in $A(D \cap \Lambda)$ that agrees with f on $\Lambda \cap bD$. In [St1] it was shown, using the complex Radon transform, that, when bD is smooth, the continuous functions on bD with the one-dimensional extension property with respect to all complex lines that meet D are precisely the functions that extend through D to members of $A(D)$. Another proof of this theorem was given by Kytmanov, who used the Bochner-Martinelli integral formula [AJ, pp. 197–198]. A local version of the result was given in [St2] and another in [Gl1]. The proofs given in [St1] and [AJ] of the results on the one-dimensional extension property do not make full use of the hypotheses; weaker assumptions are seen to suffice.

In this paper we consider a strengthening of these results that involves what we term the *Morera property*. Given a bounded domain D in $\mathbb{C}^N$ that has smooth boundary or that is convex, the continuous function f on bD is said to have the Morera property with respect to a complex k-plane Λ, $1 \leqslant k \leqslant N-1$, that intersects bD transversely, if, for each $(k, k-1)$ form β on $\mathbb{C}^N$ with constant coefficients, the integral $\int_{\Lambda \cap bD} f\beta$ vanishes. If Λ is a complex line that meets bD transversely, then f has the Morera property with respect to Λ if and only if, given a parameterization of Λ, say $z = \varphi(\zeta) = z_0 + \zeta u$ for some choice of z_0 and u in $\mathbb{C}^N$, the integral $\int_{\varphi^{-1}(bD)} f(\varphi(\zeta))d\zeta$ vanishes.

If D is a bounded domain in $\mathbb{C}^N$ with smooth boundary, then every function $f \in A(D)$ satisfies the weak tangential Cauchy-Riemann equations on bD; i.e., for every smooth $(N, N-2)$ form α on $\mathbb{C}^N$ we have $\int_{bD} f\bar{\partial}\alpha = 0$. This implies that, if $f \in A(D)$, then $f|bD$ has the Morera property with respect to each complex k-plane,

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