

# AN INVERSE IMAGE THEOREM FOR SHEAVES WITH APPLICATIONS TO THE CAUCHY PROBLEM

ANDREA D'AGNOLO AND PIERRE SCHAPIRA

## CONTENTS

0. Introduction	
1. Review on sheaves	
1.1. Geometry	453
1.2. The category $D^b(X)$	454
1.3. The category $D^b(X; p_X)$	455
2. An inverse image theorem for sheaves	
2.1. The main theorem	457
2.2. A particular case	462
3. Applications to the Cauchy problem	
3.1. Ramified solutions	464
3.2. Ramified solutions of logarithmic type	467
3.3. Decomposition at the boundary	468
References	471

**0. Introduction.** There is a wide literature on the so-called Cauchy problem. Let us recall in particular the following papers.

- (i) In 1976, Hamada, Leray, and Wagschal solved the initial value problem for a linear partial differential equation when the data are ramified along the characteristic hypersurfaces. Their proof of this result relies essentially on the precised Cauchy-Kowalevski theorem of Leray. (See [HLW].)
- (ii) In 1978, Kashiwara and Schapira proposed a new proof and an extension of the previous work to general systems, when the data are of logarithmic type. This time microdifferential operators and complex contact transformations were involved. (See [KS1].)
- (iii) In 1988, Schiltz showed how the holomorphic solution for the Cauchy problem can be expressed as a sum of functions which are holomorphic in domains whose boundary is given by the real characteristic hypersurfaces issued from the boundary of a strictly pseudoconvex domain where the data are defined. (See [Sc].)

The aim of this paper is to propose a new approach to the Cauchy problem based on sheaf theory, or more precisely, on its microlocal version. By this method we shall, in particular, recover the above results and even extend (i) and (iii) to general systems of partial differential equations (i.e., to  $\mathcal{D}$ -modules).

Received 20 March 1991. Revision received 8 June 1991.