Vol. 64, No. 2 DUKE MATHEMATICAL JOURNAL © November 1991

# THE MODULI SPACES OF SOME RANK-2 STABLE VECTOR BUNDLES OVER ALGEBRAIC $K3$-SURFACES

KANG ZUO

**0. Introduction.** An algebraic $K3$-surface is an algebraic surface $X$ with the trivial canonical line bundle $K_X \simeq \mathcal{O}_X$ and the vanishing irregularity $q(X) = 0$. Fixing an ample line bundle $H$ on $X$, for a line bundle $D$ on $X$ and an integer $k$, let $M(2, D, k)$ be the moduli space of rank-2 $H$-stable vector bundles $E$ over $X$ with $\det E = D$ and $c_2(E) = k$. It is well known that $M(2, D, k)$ is not empty if $k$ is sufficiently large. $M(2, D, k)$ is smooth and has dimension $-\chi(\mathrm{End}_0(E)) = 4k - D^2 - 6$. We are interested in the global properties of the moduli spaces. In this direction Mukai [M] has shown that the moduli spaces of simple sheaves over $X$ has a symplectic structure, i.e., a nowhere-degenerated holomorphic 2-form. A nice example of symplectic varieties is $\mathrm{Hilb}^l(X)$, the Hilbert scheme of 0-dimensional subschemes of $X$ with length $l$. This was done by Beauville [B] and Mukai [M] as an application of his main theorem.

Inspired by the above theorems, in this paper we are looking for a closer relationship between $\mathrm{Hilb}^l(X)$ and the moduli spaces of rank-2 stable vector bundles with some carefully chosen Chern classes but infinitely many of them. We prove the following statement.

THEOREM 1. *Suppose $X$ is an algebraic $K3$-surface and $H$ is an ample line bundle on $X$. Let $M(2, 0, k(n))$ be the moduli space of $H$-stable rank-2 vector bundles $E$ over $X$ with* $\det E = 0$, *$c_2(E) = k(n) := n^2H^2 + 3$, $n \in N^+$ and let* $\mathrm{Hilb}^{2k(n)-3}(X)$ *be the Hilbert scheme of 0-dimensional subschemes of $X$ with length $2k(n) - 3$. Then there is a birational map*

$$\phi\colon M(2, 0, k(n)) \simeq \mathrm{Hilb}^{2k(n)-3}(X).$$

The map $\phi$ is constructed as follows. For a generic $E \in M(0, k(n))$ we show that the sections space $H^0(X, E \otimes \mathcal{O}_X(nH))$ has only one nonzero section and that this section has only the isolated zero locus $z \in \mathrm{Hilb}^{2k(n)-3}(X)$; hence, we define the map $\phi$ by sending $E$ to $z$ and showing that $\phi$ is generically one-to-one (Lemma 3.1).

The pullback of the symplectic structure on $\mathrm{Hilb}^{2k(n)-3}(X)$ via the birational map $\phi$ gives a symplectic structure on $M(2, 0, 2k(n))$. This symplectic structure coincides with the symplectic structure given by Mukai.

**1. Preliminaries.** In this section and Section 2 we assume that $X$ is an arbitrary algebraic surface with $q(X) = 0$. Let $|L|$ be a nonempty linear system on $X$ and $z$ be a 0-dimensional subscheme of $X$ with length $l$. Consider the sublinear system

Received 21 December 1990. Revision received 15 May 1991.