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ON DEFORMATION OF NEF VALUES

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Introduction. Let L be an ample line bundle over a smooth projective variety X. It follows from a theorem of Kawamata that, if the canonical divisor K_X of X is not nef, then there exists a positive rational number τ, which we call the *nef value* of L, such that the $\mathbb{Q}$-divisor $K_X + \tau L$ is nef but not ample. From the Contraction Theorem of Kawamata and Shokurov, it follows that $K_X + \tau L$ is actually semi-ample. The map associated to the linear system $|m(K_X + \tau L)|$, for $m \gg 0$, is usually called the *adjunction morphism* and is the object of an extensive study; see, for example, [BFS] for an overview of the present state of the adjunction theory.

In the present paper we prove that the *nef value* is invariant under deformation, Theorem 1.7. This result has its roots in the theory of extremal rays on smooth manifolds (see Proposition 1.3) and is obtained by playing the Strong Lefschetz Theorem (Key Lemma 1.1) against an estimate on the locus of an extremal ray, 1.2. Examples 1.4 and 2.3 show the boundary cases for which the theory fails. As an application, in Section 2 we make some improvement upon a result of Ein (Theorem 2.4). This is a partial answer to a conjecture stated by Beltrametti and Sommese in connection with the characterization of adjunction morphisms; see 2.6 and 2.9.

The present paper was conceived when I was preparing a talk on applications of the Strong Lefschetz Theorem in the proofs of Barth-Lefschetz type results according to papers of Hartshorne, Sommese, and Lazarsfeld. I was very much influenced by these papers. The talk was prepared for the Autumn School of Algebraic Geometry which took place at Rajgród, Poland. I would like to thank the organizers of the conference for providing nice working conditions and a stimulating atmosphere.

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Section 1.

1.0. *The set up.* In the present paper all varieties are defined over the field of complex numbers. We consider a family of smooth projective varieties $\pi: X \to S$ parametrized by a smooth irreducible variety S; that is, X is a smooth variety, projective over S, and the map π is smooth so that the fibers of π are projective manifolds. We fix a closed point $0 \in S$. The variety S does not have to be complete. For example, S can be the spectrum of a discrete valuation ring or, equivalently, a

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