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ON NEVANLINNA'S ERROR TERMS

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1. Introduction. In recent years there has been considerable interest in finding the precise error term which appears in R. Nevanlinna's theory of meromorphic functions. Such questions were raised by S. Lang [4], [5], who was motivated by analogies between value-distribution theory and the approximation of real numbers by rational numbers in number theory. (See [8] and [9].) An up-to-date account of these matters appears in [5], where all terms used here are defined. Standard references for Nevanlinna theory are [2] and [7].

In this paper we give examples of $f: \mathbb{C} \to \mathbb{P}^1$ which show that the estimates in [5] are very precise. We also simplify the form of the error term function which appears in [5].

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2. Statement of results. The expressions for the error term in [5] depend on the method initiated by F. Nevanlinna and Ahlfors [7] and follow the earlier work of P. M. Wong [11]. Motivated by number-theoretic analogies, Lang calls a positive and increasing function $\psi(x)$ on $[1, \infty)$ a Khinchine function if

$$\int_1^\infty \frac{1}{x\psi(x)}\,dx = b_0(=b_0(\psi)) < \infty, \tag{2.1}$$

and such ψ's are ubiquitous in the results of [5]. We denote by $\phi(x)$ a positive increasing function in $[1, \infty)$ such that

$$\int_1^\infty \frac{1}{x\phi(x)}\,dx = \infty, \tag{2.1'}$$

and our examples will be stated for a general $\phi(x)$ as in (2.1′). Let μ be the Lebesgue measure on $1 \leqslant r \leqslant \infty$.

We state our results as three theorems. Each consists of two parts. Part (A) is a more convenient but essentially equivalent statement of the results in [5] while part (B), the main purpose of our work, shows the best nature of the corresponding part (A).

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