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CLASSIFICATION OF SOLUTIONS OF SOME NONLINEAR ELLIPTIC EQUATIONS

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0. Introduction. The work concerning the symmetry of solutions of second-order elliptic equations on an unbounded domain was first done by Gidas, Ni, and Nirenberg [2], and then generalized to infinite cylinders by Berestycki and Nirenberg [3]. Recently, Li improved their results and simplified the proofs; see [1].

In the elegant paper of Gidas, Ni, and Nirenberg [2], one of the interesting results is on the symmetry of the solutions of

$$\Delta u + u^p = 0, \qquad x \in R^N, \qquad n \geqslant 3. \tag{1}$$

They proved that for $p = (n+2)/(n-2)$ all the positive solutions of (1) with reasonable behavior at infinity, namely $u = O(|x|^{2-n})$, are radially symmetric about some point, and hence assume the form $u(x) = [n(n-2)\lambda^2]^{(n-2)/4}/(\lambda^2 + |x - x^0|^2)^{(n-2)/2}$ for $\lambda > 0$ and some $x^0 \in R^n$. This uniqueness result, as was pointed out by R. Schoen, is in fact equivalent to the geometric result due to Obata [4]: A Riemannian metric on S^n which is conformal to the standard one and having the same constant scalar curvature is the pullback of the standard one under a conformal map of S^n to itself. Recently, Caffarelli, Gidas, and Spruck [5] removed the growth assumption $u = O(|x|^{2-n})$ and proved the same result. In the case that $1 \leqslant p < (n+2)/(n-2)$, Gidas and Spruck [6] showed that the only nonnegative solution of (1) is 0.

Another similar problem of interest is the uniqueness of the solutions of the following problem

$$\begin{cases} \Delta u + \exp u = 0, & x \in R^2 \\ \displaystyle\int_{R^2} \exp u(x)\, dx < +\infty . \end{cases} \tag{2}$$

It is known that $\phi_{\lambda,x^0}(x) = \ln(32\lambda^2)/(4 + \lambda^2|x - x^0|^2)^2$, $\lambda > 0$, $x^0 \in R^2$ is a family of explicit solutions. Then one would naturally ask if these are the only solutions of (2).

In Section 1 of this paper, based on an estimate of the upper bound on the solutions at infinity, applying the method of moving planes improved in Li [1], we prove the following theorem.

THEOREM 1. *Every solution of* (2) *is radially symmetric with respect to some point in R^2 and hence assumes the form of $\phi_{\lambda,x^0}(x)$.*

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