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ON THE EQUIVALENCE RELATION FORCED BY THE SINGULARITIES OF A NONDEGENERATE SIMPLICIAL MAP

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1. Introduction. The object of this note is to investigate, in the context of a nondegenerate simplicial map $X \xrightarrow{f} M^3$ from a not necessarily locally finite simplicial complex X of dimension $\leqslant 3$ to a triangulated 3-manifold, an *equivalence relation* on X which is the smallest possible such relation, compatible with f and *killing all the singularities of* f. Actually, everything we will say in this paper stays true if we replace 3 by an arbitrary n, but the only applications of this theory so far are in dimension three. The theorem below will make things more precise. This equivalence relation denoted with $\Psi(f) \subset X \times X$ is extensively used in a number of papers by this author, in particular in the one following this note in this journal, which means that the present pages will be hopefully rather useful.

Now although X itself and its quotient $X/\Psi(f)$ will be usual simplicial complexes as far as the intermediary steps in this paper are concerned, it will be convenient to read the word "simplicial" complex in a slightly looser fashion than usual; namely, two simplexes σ, σ' can have in common not just a face but a possibly disconnected union of faces (of various dimensions). This proviso will allow us to avoid subdividing things after X will have been replaced by a quotient. The spaces we consider are really "multicomplexes" in the sense of Gromov; the intersection of two simplexes is a subcomplex of each of the two simplexes in question ([Gr]).

It will be assumed that X is connected and that it has (at most) countably many simplexes.

Our X will be endowed with the *weak topology* (i.e., $F \subset X$ is closed if and only if any $F \cap \{\text{simplex}\}$ is closed), and this makes our map f continuous.

I will denote by $\mathrm{Sing}(f) \subset X$ the set of points $x \in X$ which are such that $f|\mathrm{Star}(x)$ is *not injective*. Clearly, $\mathrm{Sing}(f)$ is a subcomplex; also, since f is simplicial outside $\mathrm{Sing}(f)$, our X is locally finite. We will also consider the subset $\Phi(f) \subset X \times X$ defined by

$$(x_1, x_2) \in \Phi(f) \Leftrightarrow fx_1 = fx_2 \in M^3;$$

in other words, $\Phi(f)$ is the *equivalence relation* on X defined by the map f.

We are also interested in equivalence relations $R \subset \Phi(f)$ which are such that, if $x \in \sigma_1$ and $y \in \sigma_2$ where σ_1 and σ_2 are two simplexes of P of the same dimension

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