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THE ASYMPTOTICS OF A LATTICE POINT PROBLEM ASSOCIATED TO A FINITE NUMBER OF POLYNOMIALS I

BEN LICHTIN

Introduction. Classical Cauchy residue calculus has been a powerful tool in a vast array of analytic number theory problems for more than a century. One of the fundamental applications has been its capability to transform functional properties of Dirichlet series

$$D(s) = \sum_{1}^{\infty} c_n e^{-s \log \lambda_n}$$

into a description of the asymptotic behavior of

$$N(x) = \sum_{\lambda_n \leqslant x} c_n, \qquad \text{as } x \to \infty.$$

On the other hand, very little attention has been directed at a similar application for Dirichlet series in several complex variables, despite the well-known work on residues over the last 30 years starting with Leray.

If

$$D(s_1, \dots, s_k) = \sum_{n_1, \dots, n_k = 1}^{\infty} c_{n_1, \dots, n_k} e^{-(s_1 \log \lambda_{n_1} + \cdots + s_k \log \lambda_{n_k})} \tag{0.1}$$

is such a series, one would like to deduce asymptotic information for

$$\mathcal{N}(x_1, \dots, x_k) = \sum_{\substack{\lambda_{n_i} \leqslant x_i \\ i = 1, \dots, k}} c_{n_1, \dots, n_k}, \qquad \text{as } (x_1, \dots, x_k) \to (\infty, \dots, \infty) \tag{0.2}$$

from functional properties of $D(s_1, \dots, s_k)$, analogous to those which are known to be useful when $k = 1$.

A few examples of "lattice point problems" will suffice to indicate why such information would be interesting.

(1) The literature treating Waring's problem has been devoted to the asymptotic behavior of

$$\#\{m \in \mathbb{N}^n : P(m) = l\}$$

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