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ROGERS'S q-ULTRASPHERICAL POLYNOMIALS ON A QUANTUM 2-SPHERE

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0. Introduction. The *continuous q-ultraspherical polynomials* were first studied by Rogers [R] at the end of the nineteenth century. He used these polynomials to obtain the Rogers-Ramanujan identities. The orthogonality of these polynomials, given by an absolutely continuous integral, was settled in the late 1970's by Askey and Wilson [AW]. They also appear in the work of Macdonald [M] on orthogonal polynomials associated with root systems. From his viewpoint, the continuous q-ultraspherical polynomials are the Macdonald polynomials of type A_1.

The continuous q-ultraspherical polynomials (Rogers's q-ultraspherical polynomials) $C_n^\lambda(x; q)$ are defined by the three-term recurrence relation

$$2x(1 - q^{n+\lambda})C_n^\lambda(x; q) = (1 - q^{n+1})C_{n+1}^\lambda(x; q) + (1 - q^{n-1+2\lambda})C_{n-1}^\lambda(x; q),$$

$$C_0^\lambda(x; q) = 1, \qquad C_1^\lambda(x; q) = 2x(1 - q^\lambda)/(1 - q).$$

In this article, we use this notation $C_n^\lambda(x; q)$ in place of $C_n(x; q^\lambda|q)$ in [AI]. It is known that they are represented by the basic hypergeometric series ${}_4\varphi_3$:

$$C_n^\lambda(x; q) = \frac{(q^{2\lambda}; q)_n}{(q; q)_n} q^{-n\lambda/2} {}_4\varphi_3\left(\begin{matrix} q^{-n}, q^{n+2\lambda}, q^{\lambda/2}e^{i\theta}, q^{\lambda/2}e^{-i\theta} \\ q^{\lambda+1/2}, -q^{\lambda+1/2}, -q^\lambda \end{matrix}; q, q\right)$$

with $x = \cos\theta$. The basic hypergeometric series ${}_{m+1}\varphi_m$ is defined by

$${}_{m+1}\varphi_m\left(\begin{matrix} a_1, \dots, a_{m+1} \\ b_1, \dots, b_m \end{matrix}; q, q\right) = \sum_{k\geqslant 0} \frac{(a_1; q)_k \cdots (a_{m+1}; q)_k}{(b_1; q)_k \cdots (b_m; q)_k (q; q)_k} x^k,$$

where $(a; q)_k = \prod_{0\leqslant i<k}(1 - aq^i)$. It is also remarked that the continuous q-ultraspherical polynomials are special cases of the Askey-Wilson polynomials but not of the q-Racah polynomials, because of their orthogonalities. See also [KR].

In this article we show that the continuous q-ultraspherical polynomials $C_n^\lambda(x; q)$ appear as the spherical functions on a quantum 2-sphere $S = S_q^2(1, 1)$ of Podleś (in the parametrization of [NM0]).

The algebra of functions $A(S)$ on the quantum 2-sphere S has a right comodule structure over the algebra of functions on the quantum group $G = SU_q(2)$. This algebra $A(S)$ is decomposed into the direct sum of irreducible $A(G)$-comodules V_j

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