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INVERSION FORMULAS FOR THE k-DIMENSIONAL RADON TRANSFORM IN REAL HYPERBOLIC SPACES

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0. Introduction. D. C. Barber and B. H. Brown introduced in [BB1], [BB2] a new kind of medical tomography technique, that of *Applied Potential Tomography* (*APT*). Their objective was to determine the conductivity a appearing in the Neumann problem

$$\begin{cases} \operatorname{div}(a \operatorname{grad} u) = 0 & \text{in } \mathbf{D}, \\ a\dfrac{\partial u}{\partial n} = \psi & \text{on } \partial\mathbf{D}, \end{cases}$$

where $\mathbf{D}$ is the unit disk in the plane, the boundary current ψ is at our disposal, and the voltage potential u can be measured on $\partial\mathbf{D}$. After linearizing around $a =$ constant, they came up with an ad hoc but reasonably accurate solution of their problem.

Analyzing this method, F. Santosa and M. Vogelius observed in [SV] that it consisted in inverting a generalized Radon transform, hence an approximate inverse could be obtained following G. Beylkin's [B]: given a generalized Radon transform R, one can systematically find a backprojection R^* and a convolution-type operator T such that

$$R^*TR = I + K,$$

where I is the identity and K is a compact operator in L^2. In fact, they showed that this algorithm reproduces the approximate inversion formula found by Barber and Brown.

Santosa raised the question of finding an *exact* inversion formula. On closer observation it was easy to see that the transform R which appears in the APT problem is simply the geodesic, or X-ray, transform in the real hyperbolic plane $\mathbf{H}^2$. In the fundamental paper [H1], S. Helgason had considered, for $1 \leqslant k \leqslant n$, the totally geodesic k-dimensional Radon transform in the real hyperbolic space $\mathbf{H}^n$ (as well as in other symmetric spaces), defining a backprojection R^* and giving, for k even, an inversion of the type

$$q(\Delta)R^*R = I, \tag{0.1}$$

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