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EXPLICIT FORMULA FOR THE SOLUTION OF CONVEX CONSERVATION LAWS WITH BOUNDARY CONDITION

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1. Introduction. We consider the mixed initial boundary value problem for strictly convex conservation laws

$$u_t + f(u)_x = 0 \tag{1.1}$$

in $x > 0$, $t > 0$, with initial condition

$$u(x, 0) = u_0(x). \tag{1.2}$$

The boundary condition is prescribed in the sense of Bardos, Leroux and Nedelec [1]. Let $u_b(t)$ be a given bounded function, then this condition requires $u(0, t)$ to satisfy the following:

$$\sup_{k \in I(u(0,t), u_b(t))} \{\mathrm{Sgn}(u(0, t) - k)(f(u(0, t)) - f(k))\} = 0 \quad \text{a.e. } t > 0,$$

where

$$I(u(0, t), u_b(t)) = [\mathrm{Min}\{u(0, t), u_b(t)\}, \mathrm{Max}\{u(0, t), u_b(t)\}].$$

When $f(u)$ is strictly convex, i.e., when $f''(u) > 0$, this condition is equivalent to (see Le Floch [3]) saying:

$$\begin{array}{ll} \text{either} & \\ \text{or} & \left.\begin{array}{l} u(0, t) = \bar{u}_b(t) \\ f'(u(0, t)) \leqslant 0 \text{ and } f(u(0, t)) \geqslant f(\bar{u}_b(t)) \end{array}\right\} \end{array} \tag{1.3}$$

where

$$\bar{u}_b(t) = \mathrm{Max}\{u_b(t), \lambda\} \tag{1.4}$$

and λ is the unique point where $f'(u)$ changes sign. Because of the strict convexity of f, f attains its minimum at λ, i.e, $f(\lambda) = \inf f(u)$.

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