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GEOMETRIC METHODS IN SCATTERING THEORY OF THE CHARGE TRANSFER MODEL

ULRICH WÜLLER

1. Introduction and results. The charge transfer model describes the behavior of an electron in the potential of N nuclei. The latter are treated classically because of their much greater mass. Their paths $q_j(t)$ are assumed to be given. Thus the quantum mechanical time evolution of the electron is generated by the Hamiltonian

$$H(t) = H_0 + \sum_{j=1}^{N} V_j(x - q_j(t)) \tag{1.1}$$

where $H_0 = -1/(2m)\Delta$. We consider only linear paths

$$q_j(t) = tv_j, \qquad v_j \in \mathbb{R}^\nu, j = 1, \ldots, N, \tag{1.2}$$

with $v_j \neq v_n$ for $j \neq n$. The following four assumptions define the class of admitted potentials $V_j = V_j(x)$.

(A I) *Every V_j is an H_0- bounded multiplication operator with relative bound zero.*

This implies the self-adjointness of $H_j = H_0 + V_j(x), j = 1, \ldots, N$, and $H(t)$, $t \in \mathbb{R}$, on $D(H_0) = W^{2,2}(\mathbb{R}^\nu)$, but not the existence of the time evolution. Hence we need the following.

(A II) *There exists a propagator for $H(t)$, i.e., a family of unitary operators $\{U(t, s)\}_{t,s \in \mathbb{R}}$ with*

(a) $U(t, t) = 1$ *for all* $t \in \mathbb{R}$,
(b) $U(t, s)U(s, r) = U(t, r)$ *for all* $t, s, r, \in \mathbb{R}$,
(c) $U(t, s)D(H_0) = D(H_0)$ *for all* $t, s, \in \mathbb{R}$,
(d) *for each* $\psi \in D(H_0)$ *and* $s \in \mathbb{R}$, *the function* $\psi(t) \equiv U(t, s)\psi$ *is a continuously differentiable* $L^2(\mathbb{R}^\nu)$*-valued function solving*

$$\frac{\partial}{\partial t}\psi(t) = -iH(t)\psi(t). \tag{1.3}$$

For potentials with H_0-bounded derivatives $\nabla V_j(x)$, the existence is well known (e.g., Theorem X.71 in [12]). The most interesting case of Coulomb potentials is treated in [11] and [14]. Using smoothing properties of the free time evolution

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