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VANISHING CYCLES ON GRASSMANNIANS

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0. Introduction. This work arose from our attempts to understand the remarkable paper of V. Ginsburg [G]. Ginsburg was interested in the following conjecture, which is very important for the representation theory.

Let X denote the flag variety associated to a complex semi-simple group G. For an element w of the Weyl group of G let $\mathscr{L}_w$ denote the irreducible $\mathscr{D}$-module with regular singularities on X supported on the Schubert variety $\overline{X_w}$. The following conjecture first appeared in an equivalent formulation in [KL].

CONJECTURE. *For $G = SL_n$, the singular support of $\mathscr{L}_w$ is an irreducible subvariety of T^*X.*

The conjecture is false for other classical groups, see e.g., computations by Tanisaki for $G = B_3, C_3$ in [T]. It is also known that the conjecture is true for $n \leqslant 6$. So far there was no general computation of the singular support for any class of Schubert varieties.

In this paper we treat the case of Grassmannians—degenerate flag varieties for SL_n—and prove the following theorem.

0.1. Theorem. *Let $G_{k,l}$ denote the Grassmannian of k-planes in $k + l$ dimensional complex vector space. Let $\overline{X_\lambda}$ be a Schubert variety in $G_{k,l}$. Let $\mathscr{L}_\lambda$ denote the irreducible holonomic $\mathscr{D}$-module with regular singularities on $G_{k,l}$ supported on $\overline{X_\lambda}$. Then the singular support of $\mathscr{L}_\lambda$ is an irreducible subvariety of $T^*G_{k,l}$.*

Remark. The flag manifold for SL_n fibers over $G_{k,n-k}$. Let π denote the projection. Then the singular support of $\pi^*\mathscr{L}_\lambda$ is irreducible. Moreover $\pi^*\mathscr{L}_\lambda$ is equal to some $\mathscr{L}_w$ if and only if the set of simple reflections s such that $ws > w$ has $n - 2$ elements.

The proof of the theorem is ultimately based on detailed analysis of the geometry of the resolutions of singularities of Schubert varieties in Grassmannians introduced by A. Zelevinsky in [Z].

The paper is organized as follows. In the following section we reduce the theorem to study of the Zelevinsky's resolutions using the formula of Brylinski for calculation of multiplicities of components of the characteristic cycle of a $\mathscr{D}$-module in terms of vanishing cycles. In the remaining portion of the paper we recall the construction of [Z] and prove the key proposition stated in the next section.

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