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ON THE GAUSSIAN MAP FOR CANONICAL CURVES OF LOW GENUS

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0. Introduction. Let C be a smooth curve of genus g, with canonical sheaf ω_C. In [W2] J. Wahl noticed that the Gaussian map ϕ: $\bigwedge^2 H^0(\omega_C) \to H^0(\omega_C^3)$, defined essentially by sending $f\,dz \wedge g\,dz$ to $(fg' - gf')\,dz^3$, occurs naturally in the study of the deformation theory of the cone over the canonical image of C. In particular, he proved there that if C lies on a K3 surface, then the cone X over C deforms to that K3 surface, and he shows that this implies that the Gaussian map ϕ cannot be surjective.

In [C-H-M], the authors (together with J. Harris) proved that for genus 10 or genus at least 12, the Gaussian map ϕ (there called the Wahl map) is surjective for general C, which fits with the fact that the general curve with these genera does not lie on a K3 surface.

In genus 11 it is known that the general curve lies on a K3 (see [M-M]). Therefore ϕ cannot be surjective; of course, since it maps a space of dimension 55 to one of dimension 50, it cannot be injective either. We began this project by asking ourselves whether this was the only case in which there are no curves C for which ϕ has maximal rank.

It is rather easy to check that for genus at most 5, i.e., for the complete intersections, the map ϕ is generically injective. We therefore began our investigations by concentrating on the cases of g between 6 and 9. In fact, we succeeded in proving, using the methods of [C-H-M], that ϕ is generically injective for g at most 8. The method failed in genus 9; however we could prove that for the general curve, the kernel of ϕ is at most one-dimensional. The second author was also able to prove that in genus 11, the cokernel of ϕ has dimension 1 for a general curve.

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