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# HOMOGENEOUS VECTOR BUNDLES AND FAMILIES OF CALABI-YAU THREEFOLDS

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**Introduction.** Calabi-Yau manifolds are compact Kähler manifolds with zero first Chern class (real coefficients). Yau's solution of the Calabi conjecture shows that an equivalent characterization is the existence of a Ricci-flat Kähler metric.

For such manifolds (cf. [1]) one can find a finite étale covering which decomposes into a product of a complex torus and simply connected factors with trivial canonical bundle. The latter are either symplectic (complex dimension $2n$, holonomy group $Sp(n)$) or special unitary (complex dimension $m \geqslant 3$, holonomy group $SU(m)$).

Thus, in dimension two, up to étale coverings, we have no choice but 2-tori and $K3$ surfaces.

In dimension three, the possibilities are not yet charted; it is an open problem whether there is an infinite or a finite number of deformation-classes of Calabi-Yau threefolds.

Motivation from string theory [21] led to the production and investigation of many examples [7, 10, 11, 24]. Physicists favored "three-generation models", i.e., Calabi-Yau threefolds with Euler number $\pm 6$, and the first examples of this kind have been given by Yau and Tian [24]. Other constructions appear in [18, 20, 22].

A large number of families of Calabi-Yau threefolds is obtained by (transversally) intersecting hypersurfaces in certain products of Fano varieties (such as projective spaces [7]), that is, by considering vanishing-loci of sections in sums of line-bundles.

In this paper we consider Calabi-Yau threefolds which are given by sections in certain homogeneous vector bundles that are not totally decomposable and we study in detail the case yielding $-6$ for the Euler number. The corresponding family has 15 moduli, consisting of simply connected Calabi-Yau threefolds which, at least generically, exhibit *a very simple structure for their cone of effective curves*; the latter, namely, is the convex hull of a *finite number of extremal rays* which are generated by *rational* curves (Theorem 6.2). This, one would surmise, might not be an isolated feature, when looking at simply connected Calabi-Yau threefolds.

We note that such expectations are compatible with recent results of Wilson [23], concerning the existence of Calabi-Yau contractions ($b_1 = 0$, $b_2 > 19$).

**§1. Homogeneous spaces of $G \approx SO(2l, C)$.** In this paragraph we review known facts (cf. [6]) and fix some notations.

Let $q$ be a nondegenerate symmetric bilinear form on $W$: a complex linear space of dimension $2l$. We use the same notation for the quadratic form associated to $q$, so that $\{q(x) = 0\}$ defines a smooth quadric $Q_{2l-2}$ in $P(W) \simeq P_{2l-1}$.

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