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EXPLICIT FORMULA FOR THE LIMIT OF A DIFFERENCE APPROXIMATION

K. T. JOSEPH AND G. D. VEERAPPA GOWDA

1. Introduction. We consider the conservation law

$$u_t + [\log(ae^u + be^{-u})]_x = 0 \tag{1.1}$$

where $a > 0$, $b > 0$, $a + b = 1$, with initial condition

$$u(x, 0) = u_0(x) \tag{1.2}$$

in the quarter plane $x > 0$, $t > 0$. From the work of Bardos, Leroux and Nedelec [1] and Dubois and Lefloch [2], it is known that one cannot prescribe $u(0, t) = \lambda(t)$ arbitrarily and hope to have a solution for (1.1) and (1.2) which satisfy this boundary condition. In fact they considered parabolic approximations of general scalar conservation laws and showed that, as the viscosity goes to zero, the limit of the approximate solution satisfies a boundary entropy inequality at the boundary. Using this fact Dubois and Lefloch [2] formulated a mixed initial boundary value problem for scalar laws. They proved existence and uniqueness results for this problem. In [3] Lefloch derived a formula for the solution under the assumption that the flux function $f(u)$ is strictly convex and $f(u)/|u| \to \infty$ as $|u| \to \infty$. In our case the flux function is $f(u) = \log[ae^u + be^{-u}]$ and $f(u)/u \to \pm 1$ as $u \to \pm\infty$.

In this paper we are interested in studying the limit of the approximate solution defined by Lax's Difference Scheme [5], for the equation (1.1) in the quarter plane. Let

$$u_k^n \simeq u(k\Delta x, n\Delta t), \qquad k = 0, 1, 2, \ldots, \qquad n = 0, 1, 2, \ldots \tag{1.3}$$

$$u_k^{n+1} = u_k^n + \frac{\Delta t}{\Delta x}[g(u_{k-1}^n, u_k^n) - g(u_k^n, u_{k+1}^n)] \tag{1.4}$$

$$u_k^0 = u_0(k\Delta x) \tag{1.5}$$

$$u_0^n = \lambda(n\Delta t) \tag{1.6}$$

with $\Delta t = \Delta x = \Delta$, and the numerical flux function g is given by

$$g(u, v) = \log[ae^u + be^{-v}]. \tag{1.7}$$

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