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CONVOLUTION ESTIMATES FOR SOME DISTRIBUTIONS WITH SINGULARITIES ON THE LIGHT CONE

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Let K^z be the family of distributions on $\mathbb{R}^{n+1}$ defined as in [1] by analytic continuation from the equation

$$K^z(x, t) = \begin{cases} (t^2 - |x|^2)_+^z/\Gamma(z+1) & \text{if } (x, t) \in \mathbb{R}^n \times \mathbb{R} \text{ and } t > 0 \\ 0 & \text{if } (x, t) \in \mathbb{R}^n \times \mathbb{R} \text{ and } t \leqslant 0 \end{cases}, \quad Re(z) > -1.$$

The purpose of this paper is to prove the following results.

THEOREM 1. *Suppose* $-n/2 \leqslant z < 0$. *If*

$$\frac{1}{p} - \frac{1}{q} = 1 + \frac{2z}{n+1} \quad \textit{and} \quad 1 + \frac{z}{n} < \frac{1}{p} < 1 + \frac{z(n-1)}{n(n+1)}, \tag{1}$$

then the inequality

$$\|K^z * f\|_q \leqslant C(p)\|f\|_p \tag{2}$$

holds for measurable f *on* $\mathbb{R}^{n+1}$.

The conditions (1) of Theorem 1 appear to be necessary for (2) to hold—we comment on this in §3. A more interesting problem is whether the analogue of Theorem 1 holds for $-(n+1)/2 < z < -n/2$.

If σ denotes Lebesgue measure on the unit sphere Σ_{n-1} in $\mathbb{R}^n$, then $K_{-1} * f(x, t)$ is a multiple of

$$\int_0^\infty \int_{\Sigma_{n-1}} f(x - r\sigma, t - r)\, d\sigma\, r^{n-2}\, dr = \int_{\mathbb{R}^n} f(x - y, t - |y|)|y|^{-1}\, dy \doteq Tf(x, t).$$

Thus the next result is a special case of Theorem 1.

COROLLARY 2. *If*

$$\frac{1}{p} - \frac{1}{q} = \frac{n-1}{n+1} \quad \textit{and} \quad \frac{n-1}{n} < \frac{1}{p} < \frac{n^2+1}{n^2+n},$$

then the operator T *maps* $L^p(\mathbb{R}^{n+1})$ *into* $L^q(\mathbb{R}^{n+1})$.

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