Vol. 59, No. 3 DUKE MATHEMATICAL JOURNAL © December 1989

TANGENTIAL POLYNOMIALS AND ELLIPTIC SOLITONS

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Introduction. In the mid seventies, several people succeeded in integrating different nonlinear partial differential equations by algebro-geometric methods. It was shown how to associate a class of solutions of the Korteweg-de Vries equation:

$$\text{(KdeV)} \qquad \frac{\partial}{\partial t}u + \frac{1}{4}\left(6u\frac{\partial}{\partial x}u - \frac{\partial^3}{\partial x^3}u\right) = 0$$

to any hyperelliptic Riemann Surface ([MK-VM], [L], [D-M-N]). This was later generalized by Krichever ([K-1], see also [S-W] ch. 6) who associated a class of solutions of the Kadomtsev-Petviashvilii equation:

$$\text{(KP)} \qquad \frac{3}{4}\frac{\partial^2}{\partial y^2}u + \frac{\partial}{\partial x}\left\{\frac{\partial}{\partial t}u + \frac{1}{4}\left(6u\frac{\partial}{\partial x}u - \frac{\partial^3}{\partial x^3}u\right)\right\} = 0$$

to any integral projective curve (i.e., any compact Riemann surface with at most a finite number of singularities). At the same time, following earlier work of Calogero ([C]) there began a systematic study of the rational and doubly periodic solutions of the K-P equation. These solutions are called, respectively, rational and elliptic solitons and have the type:

$$u(x, y, t) = 2\sum_{i=1}^{n} \mathscr{P}(x - x_i(y, t)),$$

$\mathscr{P}$ being the Weierstrass function associated to Λ, the lattice of periods of the soliton u (in the rational case $\mathscr{P}(x) = 1/x^2$). Once we fix Λ, the number n is uniquely determined by u and is called the degree of the soliton u. As suggested by Kruskal ([Kru]) the motions of the poles of u were completely identified with the Hamiltonian flows of a system of n particles on the line, having as Hamiltonian function ([C], [M], [A-MK-M], [O-P], [K-2]):

$$\mathrm{H} = \frac{1}{2}\sum_{i=1}^{n} p_i^2 + 2\sum_{i\neq j} \mathscr{P}(x_i - x_j)$$

Received November 28, 1988. Revision received April 4, 1989.