Vol. 59, No. 2 DUKE MATHEMATICAL JOURNAL © October 1989

FANNING CURVES OF LAGRANGIAN MANIFOLDS AND GEODESIC FLOWS

SHAHLA AHDOUT

0. Introduction. In this paper we consider curves $\alpha(s)$, $-t < s < t$, of Lagrangian submanifolds of a symplectic manifold (M, ω) passing through a fixed point $\bar{p}$ and study its local properties near $\bar{p}$. When a nondegeneracy condition is satisfied, the curve is called *fanning*; in this case, we parametrize the curve by a time-dependent function on $\alpha(0)$ whose Taylor series coefficients in t are symplectic invariants of the curve (Theorem 2.6).

An example of such a curve can be obtained in the cotangent bundle of a Riemannian manifold from the geodesic flow. Using this curve, we prove in Theorem 4.3 that the curvature tensor is a symplectic invariant of the geodesic flow.

The second example of this type of curve is obtained from the billiard ball map on a compact, strictly convex domain in $\mathbb{R}^{n+1}$. The study of symplectic invariants of this curve is of interest in the inverse obstacle scattering problem (see [7]) and in the inverse spectral problem (see [5]). In this paper, using these invariants, we show that up to a motion on $\mathbb{R}^{n+1}$, strictly convex hypersurfaces of $\mathbb{R}^{n+1}$ can be locally distinguished by their symplectic invariants (Theorem 5.5). This result increases our hope that the following conjecture of Guillemin and Melrose is true (see [2]):

(i) *Two compact, strictly convex regions in $\mathbb{R}^{n+1}$ with the same Dirichlet spectrum have symplectomorphic billiard ball maps.*
(ii) *Two regions with symplectomorphic billiard systems are the same up to a rigid motion of $\mathbb{R}^{n+1}$.*

A brief summary of the contents of this paper follows.

In section 1 we study a curve $\alpha(s)$, $-t < s < t$, of Lagrangian subspaces of a symplectic vector space (V, ω). The set $\Lambda(V)$ of Lagrangian subspaces of V is a nonsingular algebraic subvariety of the Grassmann manifold of n-planes in V; $\mathrm{sym}(\alpha(0))$, the set of symmetric bilinear forms on $\alpha(0) = L_0$, is a local coordinate system of $\Lambda(V)$. If $\alpha(s)$ is parametrized by $b(s) \in \mathrm{sym}(L_0)$, then $\alpha(s)$ is said to be fanning if $\dot{b}(0)$ is nondegenerate. In this case, a parametrization for α is given in Theorem 1.3. This also can be realized as a curve in the homogeneous space G/H, where $G = SP(V)$ and $H = \{g \in G: g(L_0) = L_0\}$. Then, using the idea of moving frames, we can obtain the invariants of α (see [1]).

Received October 13, 1988. Revision received March 18, 1989.