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ON ISOSPECTRAL LOCALLY SYMMETRIC SPACES AND A THEOREM OF VON NEUMANN

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1. Introduction. Call two lattices Γ_1 and Γ_2 in a locally compact group G *isospectral* if the representations of G on $L^2(G/\Gamma_1)$ and $L^2(G/\Gamma_2)$ are unitarily equivalent. Here we show that isospectral lattices are quite common in semisimple real algebraic groups $\mathbf{G}$. We construct them using a variation of a technique of Sunada [Su]. As a consequence, we obtain new examples of locally symmetric spaces with isospectral Laplacians. In particular, we get such examples with higher rank. Previously, the only higher-rank isospectral manifolds known were locally reducible. These were Vigneras's examples of quotients of products of hyperbolic spaces [V]. On the other hand, Brooks found irreducible higher-rank locally symmetric spaces with isospectral potentials [Br]. However, the underlying manifolds in his examples happened to be isometric. This was due to Mostow rigidity, since the fundamental groups were isomorphic. In fact, this work was largely motivated by the question of whether the rigidity properties of lattices in higher rank force an isospectral rigidity.

THEOREM 1.1. *Let $\mathbf{G}$ be a noncompact almost simple connected real algebraic group whose complexification is of one of the following types:*

(a) A_n *with* $n \geqslant 26$
(b) C_n *with* $n \geqslant 27$
(c) B_n *or* D_n *with* $n \geqslant 13$

Then any cocompact lattice in $\mathbf{G}$ contains nonisomorphic isospectral torsion-free subgroups of finite index.

We believe that our rank condition is far from optimal. Note that if we fix a maximal compact subgroup K of $\mathbf{G}$, then the locally symmetric spaces $K\backslash\mathbf{G}/\Gamma_i$ defined by Γ_1 and Γ_2 are manifolds as the Γ_i are torsion-free. Since the spectrum of the Laplacian is given by the spectrum of the Casimir element on the K-fixed vectors of $L^2(\mathbf{G}/\Gamma)$, we immediately obtain the geometric:

COROLLARY 1.2. *Let M be a compact locally symmetric space of the noncompact type. Assume that the isometry group of the universal cover of M is as in Theorem 1.1. Then M is finitely covered by two nonisometric isospectral symmetric spaces.*

This corollary yields new examples even in real rank 1, in particular for hyperbolic space.

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