Vol. 59, No. 1 DUKE MATHEMATICAL JOURNAL © August 1989

SEMIRIGID PARTIAL DIFFERENTIAL OPERATORS AND MICROLOCAL ANALYTIC HYPOELLIPTICITY

A. ALEXANDROU HIMONAS

1. Introduction and main results. This paper gives new microlocal analytic hypoellipticity results for a class of partial differential operators of higher order. These results extend in a certain direction the recent work of Baouendi-Rothschild [1] dealing with systems of vector fields. Let $P = P(y, D)$ be a linear partial differential operator (p.d.o.) of order $m \geqslant 1$ with complex-valued real-analytic coefficients defined in an open neighborhood Ω of $y_0 \in R^n$. Denote by $P_m(y, \eta)$ its principal symbol and set

$$p_1(y, \eta) = \operatorname{Re} P_m(y, \eta) \quad \text{and} \quad p_2(y, \eta) = \operatorname{Im} P_m(y, \eta). \tag{1.1}$$

Let $\gamma = (y_0, \eta_0) \in T^*\Omega - \{0\}$ be a characteristic point for P and assume that the operator P is of principal type at γ i.e.,

$$P_m(\gamma) = 0 \quad \text{and} \quad \operatorname{grad}_\eta P_m(\gamma) \neq 0. \tag{1.2}$$

If $f(y, \eta)$ and $g(y, \eta)$ are smooth functions defined on $T^*\Omega - \{0\}$, we denote their Poisson bracket by

$$\{f, g\} = \sum_{j=1}^{n} \left(\frac{\partial f}{\partial \eta_j}\frac{\partial g}{\partial y_j} - \frac{\partial f}{\partial y_j}\frac{\partial g}{\partial \eta_j}\right)$$

and the Hamiltonian of f, H_f, by

$$H_f = \sum_{j=1}^{n} \left(\frac{\partial f}{\partial \eta_j}\frac{\partial}{\partial y_j} - \frac{\partial f}{\partial y_j}\frac{\partial}{\partial \eta_j}\right).$$

Note that $-H_g f = H_f g = \{f, g\}$.

The p.d.o. P is said to be of *finite type at* γ (see Egorov [5], [6]; Hörmander [9], [10]; Kohn [11]) if

$$\{p_{i_j}, \{p_{i_j-1}, \ldots, \{p_{i_2}, p_{i_1}\} \cdots\}\}(\gamma) \neq 0, \tag{1.3}$$

for some $(i_1, \ldots, i_j)$, where $i_1, \ldots, i_j$ are either 1 or 2. If $k = k(\gamma)$ is the smallest such j, then P is said to be of *type* k at γ. For example, $k(\gamma) = 1$ iff P is elliptic at γ; $k(\gamma) = 2$ iff $p_1(\gamma) = p_2(\gamma) = 0$ and $\{p_1, p_2\}(\gamma) \neq 0$.

Received November 5, 1987. Revision received January 18, 1989. Partially supported by the NSF.