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WIENER'S CRITERION FOR DIVERGENCE FORM PARABOLIC OPERATORS WITH C^1-DINI CONTINUOUS COEFFICIENTS

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1. Introduction. A long-standing open problem in parabolic potential theory has been that of the convergence of the solution of the Dirichlet problem to the assigned boundary values. For Laplace operator, such a question was settled by Wiener in his celebrated 1924 paper [W]. In 1982 Evans and Gariepy [EG] were able to extend Wiener's criterion to the heat operator $H = \Delta - D_t$ in $\mathbf{R}^{n+1}$. Recently, Garofalo and Lanconelli [GL1] have proved Wiener's criterion for a general second-order parabolic operator with C^∞ coefficients.

In this paper we take up the study started in [GL1]. Our aim is to prove Wiener's criterion for operators similar to those considered in [GL1], but with minimal smoothness assumptions on the coefficients. More specifically, we consider operators in $\mathbf{R}^{n+1}$ of the type

$$L = \operatorname{div}(A(x, t)\nabla_x) - D_t, \tag{1.1}$$

where $(x, t) \mapsto A(x, t) = (a_{ij}(x, t))$ is a real, symmetric, matrix-valued function on $\mathbf{R}^{n+1}$ such that there exists $\lambda \in (0, 1]$ for which

$$\lambda|\xi|^2 \leqslant \sum_{i,j=1}^{n} a_{ij}(x, t)\xi_i\xi_j \leqslant \lambda^{-1}|\xi|^2 \tag{1.2}$$

for every $(x, t) \in \mathbf{R}^{n+1}$ and every $\xi \in \mathbf{R}^n$. We also assume that the entries of the matrix A are *C^1-Dini continuous.* In order to explain what we mean by this, we set some notation. Henceforth in this paper the letters z, ζ, z_0, ζ_0 will respectively denote the points $(x, t), (\xi, \tau), (x_0, t_0), (\xi_0, \tau_0)$ in $\mathbf{R}^{n+1}$. Then for $z \in \mathbf{R}^{n+1}$ we let $\|z\|^2 = |x|^2 + |t|$, where $|x|^2 = \sum_{j=1}^n x_j^2$. We say that the matrix-valued function $z \mapsto A(z)$ is C^1-Dini continuous if there exists an increasing function ω: $[0, +\infty) \to [0, +\infty)$, called the *$C^1$-Dini modulus of continuity of A*, such that

$$\int_0^1 \omega(\delta)\frac{d\delta}{\delta} < +\infty, \tag{1.3}$$

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