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GENERIC TORELLI THEOREM FOR HYPERSURFACES OF CERTAIN COMPACT HOMOGENEOUS KÄHLER MANIFOLDS

Dedicated to Professor Ichiro SATAKE on the occasion of his sixtieth birthday.

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Introduction. The generic Torelli problem asks whether the period map is generically injective. In 1980, Carlson-Griffiths [5] introduced the notion of the infinitesimal variations of Hodge structure (abbreviated as IVHS) and opened a way to attack this problem by showing the generic Torelli theorem for cubic hypersurfaces in $\mathbb{P}^{3m+1}$. It was Donagi [7] who succeeded in extending their result to projective hypersurfaces of almost all degree, developing useful techniques such as the symmetrizer lemma. His method is quite applicable to other cases: Sufficiently ample hypersurfaces of a projective manifold with the very ample canonical bundle ([12]), and certain weighted projective hypersurfaces ([9] and [24]).

In this article, we consider the generic Torelli problem for hypersurfaces of a simply-connected compact homogeneous Kähler manifold, that is, a *Kähler C-space* [30], whose second Betti number equals one. Since an irreducible Hermitian symmetric space of compact type (e.g., a Grassmannian) is necessarily a Kähler C-space with $b_2 = 1$, ours can be considered as a natural extension of projective hypersurfaces. Let Y be a Kähler C-space with $b_2(Y) = 1$. Then the Picard group of Y is isomorphic to $\mathbb{Z}$ and we denote by $\mathcal{O}_Y(1)$ its ample generator. A smooth member X of the linear system $|\mathcal{O}_Y(d)|$ will be called a smooth hypersurface of degree d.

Our observation will go along the analogous line as in the case of projective hypersurfaces in [7], which can be roughly illustrated as follows: (1) Study of the Jacobian ring R of X (the duality theorem), (2) Interpretation of IVHS of X by R, (3) The symmetrizer lemma, (4) Recovering X from its IVHS. Let us comment on each step to clarify the problem we shall encounter. In [12], Green defined the Jacobian ring of a smooth hypersurface of an arbitrary variety and gave cohomological criteria to assure its duality properties. Also, some cohomological conditions are known for (2) (cf. [23]). Thus our task in the steps (1) and (2) is reduced to checking these conditions. In fact, if Y is a projective space, one can use a well-known theorem of Bott [4]. In the general case, however, the lack of knowledge about $H^q(\Omega_Y^p(a))$ is quite serious. In the steps (3) and (4), we need to know the degree of the generators of the defining ideal of the "Veronese images" of Y. If Y is a projective space, it is known that the ideal in question is generated by quadrics. But in our case, we do not know.

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