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TRANSCENDENCE AND DRINFELD MODULES: SEVERAL VARIABLES

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0. Introduction.

Notations.

$\mathbb{F}_q$ the finite field with q elements, q a power of p
$\mathscr{C}$ smooth projective geometrically irreducible curve over $\mathbb{F}_q$
∞ a closed point of $\mathscr{C}$ with degree d_∞
k the function field of $\mathscr{C}$ over $\mathbb{F}_q$
A the ring of functions in k regular on $\mathscr{C} - \{\infty\}$
k_∞ the completion of k at ∞
$\bar{k}_\infty$ the algebraic closure of k_∞
$\mathbb{N}$ the set of nonnegative integers

In this article, we shall extend our previous work in [13]. The central theme is to explore further the analogy between number fields and global function fields, in the direction of transcendence theory, with Drinfeld modules (or, more generally, abelian t-modules) playing the role of abelian varieties.

Let φ_L be a Drinfeld A-module defined over $\bar{k}$, with associated exponential function $e_L(z)$ and period lattice $L \subset \bar{k}_\infty$. Let K_L be the field of multiplications of ϕ_L. We shall prove the following analogue of the qualitative form of Baker's theorem on logarithms of algebraic numbers:

THEOREM 0.1. *Let $\alpha_1, \ldots, \alpha_n$ be elements of $\bar{k}_\infty$ such that $e_L(\alpha_i)$ are in $\bar{k}$ for $i = 1, \ldots, n$. If $\alpha_1, \ldots, \alpha_n$ are linearly independent over K_L, then any linear combination of $\alpha_1, \ldots, \alpha_n$, with coefficients not all zero from the separable closure K_L^s, is transcendental over k.*

We shall say an element in $\bar{k}_\infty$ is transcendental if it is transcendental over k.

We also study the transcendence properties of Hilbert-Blumenthal-Drinfeld modules, based on the algebraic foundation provided by G. Anderson [1]. The Hilbert-Blumenthal-Drinfeld modules are higher-dimensional Drinfeld modules, which are analogues of the Hilbert-Blumenthal abelian varieties, i.e., abelian varieties with sufficiently many real endomorphisms. We shall prove that if a Hilbert-Blumenthal-Drinfeld module is defined over $\bar{k}$, then its periods have all coordinate components transcendental. This is also parallel to the classical situation; cf. Bertrand [3].

The above-mentioned results are proved by adapting the method of Schneider-Lang-Bertrand-Masser to characteristic p. The main theorem in §2 can be viewed

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