

# ARITHMETIC CLASSIFICATION OF KUGA FIBER VARIETIES OF QUATERNION TYPE

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*Dedicated to Professor Michio Kuga on the occasion of his sixtieth birthday*

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**0. Introduction.** A Kuga fiber variety of quaternion type  $A \xrightarrow{f} V$  is a fiber space whose fibers are abelian varieties and which is parametrized by a Hilbert modular variety  $V = \Gamma \backslash \mathcal{H}^t$ .

The construction of  $\Gamma \backslash \mathcal{H}^t$  is classical. Let  $k$  be a totally real algebraic number field and  $B$  a quaternion algebra over  $k$ . Then we have an isomorphism

$$B \bigotimes_{\mathbb{Q}} \mathbb{R} \cong M_2(\mathbb{R}) \times \cdots \times M_2(\mathbb{R}) \times \mathbb{K} \times \cdots \times \mathbb{K},$$

where  $M_2(\mathbb{R})$  is the total matrix algebra of degree two and  $\mathbb{K}$  the algebra of real quaternions. Let  $t > 0$  be the number of copies of  $M_2(\mathbb{R})$  and  $G = \text{Res}_{k/\mathbb{Q}}\{a \in B \mid aa^i = 1\}$ , where  $i$  is the canonical involution.

Any arithmetic subgroup  $\Gamma$  of  $G$  is a discontinuous group of transformations of the product of upper half planes  $\mathcal{H}^t$ . It defines an arithmetic variety  $\Gamma \backslash \mathcal{H}^t$ .

A Kuga fiber variety over  $\Gamma \backslash \mathcal{H}^t$  is constructed from a symplectic representation of  $G$  [K], [S], [Ad1], [Ad2]. In this paper we will consider only families of Hodge

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