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ARITHMETIC CLASSIFICATION OF KUGA FIBER VARIETIES OF QUATERNION TYPE

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Dedicated to Professor Michio Kuga on the occasion of his sixtieth birthday

CONTENTS

0. Introduction. A Kuga fiber variety of quaternion type $A \xrightarrow{f} V$ is a fiber space whose fibers are abelian varieties and which is parametrized by a Hilbert modular variety $V = \Gamma\backslash\mathscr{H}^t$.

The construction of $\Gamma\backslash\mathscr{H}^t$ is classical. Let k be a totally real algebraic number field and B a quaternion algebra over k. Then we have an isomorphism

$$B \bigotimes_{\mathbf{Q}} \mathbf{R} \cong M_2(\mathbf{R}) \times \cdots \times M_2(\mathbf{R}) \times \mathbf{K} \times \cdots \times \mathbf{K},$$

where $M_2(\mathbf{R})$ is the total matrix algebra of degree two and $\mathbf{K}$ the algebra of real quaternions. Let $t > 0$ be the number of copies of $M_2(\mathbf{R})$ and $G = \mathrm{Res}_{k/\mathbf{Q}}\{a \in B \mid aa^i = 1\}$, where i is the canonical involution.

Any arithmetic subgroup Γ of G is a discontinuous group of transformations of the product of upper half planes $\mathscr{H}^t$. It defines an arithmetic variety $\Gamma\backslash\mathscr{H}^t$.

A Kuga fiber variety over $\Gamma\backslash\mathscr{H}^t$ is constructed from a symplectic representation of G [K], [S], [Ad1], [Ad2]. In this paper we will consider only families of Hodge

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