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SINGULAR RADON TRANSFORMS AND OSCILLATORY INTEGRALS

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1. Introduction. The purpose of this paper is to provide an alternative reduction of singular Radon transforms to certain oscillatory integrals. We shall do this in a way which differs from the approach we used in [PS], and in fact has two advantages over the previous method. First, for operators in $\mathbb{R}^{n+1}$, the method also works for $n = 1$, as opposed to that of [PS] which was limited to $n \geqslant 2$. Secondly, it is in principle applicable to a wider class of Fourier integral operators with singular symbols. Let us explain these matters in greater detail.

Let Ω be a compact manifold of dimension $n + 1$, and let there be given for each point P of Ω a hypersurface Ω_P passing through P. To a function f on Ω a singular Radon transform will associate the function whose value at P is the integral of f against some given density on Ω_P with a Calderón-Zygmund singularity at P. The distributions of hypersurfaces studied in [PS] are assumed to satisfy a condition of nonvanishing rotational curvature, which is a nondegeneracy condition.

In a suitably localized coordinate system (with (x, t), $(y, s) \in \mathbb{R}^{n+1}$, x, $y \in \mathbb{R}^n$, t, $s \in \mathbb{R}$), a singular Radon transform R can be written as

$$R(f)(x, t) = \int_{\mathbb{R}^{n+1}} \delta(s - t - S(x, y, t))K(x, t; x - y)f(y, s)\, dy\, ds \tag{1.1}$$

where δ is the Dirac delta function, and S satisfies (by virtue of "rotational curvature") the crucial condition that $\det\left(\dfrac{\partial^2 S}{\partial x_i \partial y_j}\right) \neq 0$. Also for each x, t, $K(x, t; z)$ is a standard singular integral kernel in $z \in \mathbb{R}^n$, with compact support.

The analysis in [PS] proceeded by expressing R as a pseudo-differential operator:

$$Rf = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\lambda t} a(t, \lambda)\hat{f}(\lambda)\, d\lambda, \tag{1.2}$$

where f took its values in $L^2_x(\mathbb{R}^n)$, and the symbol $a(t, \lambda)$ was for each (t, λ) an operator on $L^2_x(\mathbb{R}^n)$ to itself—in fact an oscillatory integral operator. The main difficulty was then to verify that a belonged to the symbol class $S^0_{1/2, 1/2}$, and here the critical problem was that the required differential inequalities for this class held only up to a certain order (depending on the dimension n), which was not sufficient

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