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HOMOLOGICAL PROPERTIES OF PERIODIC HOMEOMORPHISMS OF 4-MANIFOLDS

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Given a mapping f from a space X into itself, it is often possible to obtain significant information about f from the algebraic endomorphisms induced by f on the homology and cohomology of X. For example, if X is a compact polyhedron or topological manifold, then the Lefschetz fixed-point theorem relates the existence of fixed points for f to a function of the eigenvalues of the rational homology or cohomology self-maps defined by f (i.e., the *Lefschetz number*; compare [G-H]). Frequently, some natural assumptions on f and X allow one to retrieve much more information about f than in the general case. In particular, if X is a compact differentiable manifold and f is a diffeomorphism such that $f^N = 1_X$ for some X (in other words, if f is *periodic*), then the Lefschetz number of f equals the Euler characteristic $\chi(F)$ or the set of points F left fixed by f (compare [Kob]). Furthermore, if $f \neq 1$ but $f^p = 1$ for some prime p, then the action of f on the homology groups $H_k(X; \mathbf{Z})$ makes the latter into $\mathbf{Z}[\mathbf{Z}_p]$-modules, and results of R. Swan [Sw1] imply strong restrictions on these modules. For example, if X is an $(n-1)$-connected $2n$-manifold ($n \geqslant 2$) and $\xi \in \mathbb{C}$ is a primitive pth root of 1, then the $\mathbf{Z}[\xi]$-module

$$H_n(X; \mathbf{Z}) \otimes_\Lambda \mathbf{Z}[\xi]$$

(where $\Lambda = \mathbf{Z}[\mathbf{Z}_p]$) is projective and determines the zero element of the projective class group $\tilde{K}_0(\mathbf{Z}[\xi]) \cong \tilde{K}_0(\mathbf{Z}[\mathbf{Z}_p])$.

In [E2] A. Edmonds considers the extent to which such relationships hold for periodic homeomorphisms that are not necessarily smooth. The results in [E2] lead naturally to several basic conjectures and problems that are formulated throughout the article. In this paper we answer three of these questions.

Our first result is a Lefschetz formula for periodic homeomorphisms of 4-manifolds:

THEOREM 1. *Let M^4 be a closed 4-manifold, and let $f\colon M^4 \to M^4$ be a periodic homeomorphism. Then the fixed set F of f has finitely generated Čech cohomology, and the Lefschetz number of f is equal to the (Čech) Euler characteristic $\chi(F)$.*

This result does not generalize to periodic homeomorphisms on n-manifolds for $n > 4$ if the period of f is divisible by distinct primes. Classes of examples are discussed at the end of section 1.

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