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ISOSPECTRAL SETS OF CONFORMALLY EQUIVALENT METRICS

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Let M be a compact manifold. Two Riemannian metrics g and g' on M are said to be isospectral if the spectrum of the Laplacian is the same for the metrics g and g'. A standard question is to understand the extent to which the spectrum of g determines g. There are by now many constructions (see [Su] and [Br-T]) of isospectral metrics. Nonetheless, one expects isospectral metrics to be somewhat rare.

This problem was considered in two dimensions by Osgood, Phillips, and Sarnak ([O-P-S]), who showed that isospectral sets of metrics are compact in the $\mathscr{C}^\infty$ topology on the space of all metrics. Their technique made use of the determinant of the Laplacian, as well as the better-known heat invariants, which are all spectral invariants of a metric.

In the present paper, we study compactness questions in dimensions three and above. We will consider isospectral sets of conformally equivalent metrics—that is to say, we will fix a metric g on M and look for isospectral sets of metrics of the form $g' = f \cdot g$, where f is a positive function.

Our first result shows that one may still obtain nontrivial isospectral sets in this manner:

THEOREM 1. *There exists manifolds M with conformally equivalent metrics g and g' such that g is isospectral to g'.*

The method of proof of Theorem 1 is quite general and allows a great deal of freedom in the selection of M, g, and g'. It is based on the method of Sunada [Su] for constructing isospectral manifolds and is similar to the construction of [Br]. We observe that, in dimension 2, the isospectral metrics on a surface of genus 3 given in [Br-T] are conformally equivalent.

To state our main result, let us denote by $\{g\}$ the conformal class of g—that is, the space of all metrics conformally equivalent to g—and let us say that $\{g\}$ is negative if there is a metric in $\{g\}$ of negative scalar curvature.

THEOREM 2. *Let M be a compact 3-manifold and $\{g\}$ a negative conformal class on M. Then for $g' \in \{g\}$, the set of metrics in $\{g\}$ isospectral to g' is compact in the $\mathscr{C}^\infty$ topology.*

We remark that any manifold of dimension greater than 2 carries metrics of negative scalar curvature, so that our theorem involves no restriction on the topology of M.

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