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ON VARIETIES ISOMORPHIC IN CODIMENSION ONE TO TORUS EMBEDDINGS

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§1. Introduction. This paper has two purposes. One is to prove the following result:

THEOREM 1. *Let T be an algebraic torus. Suppose that X is a normal variety isomorphic in codimension one to a torus embedding–i.e., X less a subset of codimension at least two is isomorphic to an open subset of a torus embedding. Then X itself is an open subset of a torus embedding.*

COROLLARY 2. *Suppose in addition that X is complete. Then X is a torus embedding.*

The methods used to prove this result appear to apply more generally, but there will be some difficulties involving the divisor class group that must first be overcome. Later, I hope to use these methods to provide a precise description of the minimal embedded resolutions of E_8: $x^2 + y^3 + z^5 = 0$.

The proof of Theorem 1 uses some new techniques. The other purpose of this paper is to give a description in general terms of these devices. It so happens that the method is best understood in general terms, so we will present it first and then prove the theorem as an application.

Our result has some relevance to Hironaka's factorization problem. The papers of Crauder, Danilov, Moishezon, Schaps, and Teicher [1, 3, 6, 8, 9, 11] contain many results relevant to this problem. The author is grateful to Bruce Crauder for helpful correspondence and conversation.

The survey paper [2] of Danilov contains the standard results on torus embedding that we use.

§2. The method described. Throughout this section we fix a field of rational functions K which has a model X.

There is a difficulty involved in the separation of the points on a variety which has been defined by patching, which we will postpone by first considering affine varieties only. An affine variety is determined by its coordinate ring, to which our attention now turns. Recall the valuative criteria for an integral closure [7, Theorem 11.12]:

PROPOSITION 3. *A rational function g is regular on a normal variety X if and only if its Weil divisor (g) is effective.*

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