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THE DISC MULTIPLIER

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In memory of my friend J. L. Rubio

In his celebrated paper [5] C. Fefferman disproved a famous conjecture in harmonic analysis. He proved that the operator T defined by the formula $\widehat{Tf}(y) = \chi_B(y)\hat{f}(y)$ is bounded in $L^p(\mathbb{R}^n)$, $n \geqslant 2$, if and only if $p = 2$, where χ_B is the characteristic function of the unit ball and $\hat{f}(y) = \int_{\mathbb{R}^n} \exp(-ix \cdot y) f(x)\, dx$ denotes the Fourier transform. Fefferman's result came as a surprise because it was previously believed that T should be bounded on $L^p(\mathbb{R}^n)$ for $2n/(n+1) < p < 2n/(n-1)$. This operator, the disc multiplier, is important for understanding the spherical convergence of Fourier series and integrals in several variables, where it plays the same role as the classical Hilbert transforms in one-dimensional Fourier analysis. We recommend reference [4] for a rather complete exposition of this and other closely related problems.

In this paper we prove a positive result for the operator T in that range of L^p-spaces if we substitute the space $L^p(\mathbb{R}^n)$ by a mixed norm space, namely $L^p_{\text{rad}}(L^2_{\text{ang}})(\mathbb{R}^n)$, where the L^p-norm is taken in the radial variable and the L^2-norm with respect to the angular part, that is,

$$\|f\|_{p,2} = \left[\int_0^\infty \left[\int_{S^{n-1}} |f(r\bar{y})|^2\, d\sigma(\bar{y})\right]^{p/2} r^{n-1}\, dr\right]^{1/p}.$$

In his Ph.D. thesis [7] Luis Vega analyzed how the restriction theorem of the Fourier transform to circles can be transplanted to obtain higher-dimensional results in mixed norm spaces. José Luis Rubio extends that program to the Bochner-Riesz multipliers T_α, $\alpha > 0$, defined by the formula $\widehat{T_\alpha f}(\xi) = (1 - |\xi|^2)_+^\alpha \hat{f}(\xi)$. In his beautiful work entitled "Transference Principles for Radial Multipliers," also in this issue, he shows that in R^n, n even, T_α, $\alpha > 0$, is a bounded operator on $L_{p,2}$ iff $2n/(n+1+2\alpha) < p < 2n/(n-1-2\alpha)$. The transference argument does not seem to yield the proof of the cases n odd or $\alpha = 0$, which are also conjectured in that paper. However, the case $\alpha = 0$ is the relevant one, since it is best possible and implies the result for T_α, $\alpha > 0$, in any dimension where one nevertheless expects a better result to hold.

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