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TWISTED DIFFERENCE OPERATORS AND PERTURBED CHEBYSHEV POLYNOMIALS

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1. Introduction. The Chebyshev polynomials T_ν satisfy the recurrence equation

$$T_{\nu+2}(x) - 2xT_{\nu+1}(x) + T_\nu(x) = 0;$$

see section 5 for more details and explanations. It is expected that a perturbation of these polynomials will satisfy the approximate equation

$$p_{\nu+2}(x) - 2xp_{\nu+1}(x) + p_\nu(x) \approx 0. \tag{1.1}$$

In fact, something like this will indeed be true for $-1 \leqslant x \leqslant 1$ for the kind of perturbations that will be described in section 5. Introducing the difference operator

$$\Delta(t)q_\nu = e^{it}q_{\nu+1} - q_\nu \tag{1.2}$$

(caution: in section 6, $\Delta(t)$ will be defined differently; this is only a special case), the left-hand side of (1.1) can be written as

$$\Delta(\theta)\Delta(-\theta)p_\nu(x) = e^{i\theta}\big(e^{-i\theta}p_{\nu+2}(x) - p_{\nu+1}(x)\big) - \big(e^{-i\theta}p_{\nu+1}(x) - p_\nu(x)\big),$$

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