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REGULARITY OF SOLUTIONS TO THE SCHRÖDINGER EQUATION

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Let f belong to the Schwartz space $\mathscr{S}(\mathbb{R}^n)$ and set

$$S_t f(x) = u(x,t) = \int_{\mathbb{R}^n} e^{ix\cdot\xi} e^{it|\xi|^2} \hat{f}(\xi)\, d\xi, \qquad x \in \mathbb{R}^n, \qquad t \in \mathbb{R}.$$

Here $\hat{f}$ denotes the Fourier transform of f, defined by

$$\hat{f}(\xi) = \int_{\mathbb{R}^n} e^{-i\xi\cdot x} f(x)\, dx.$$

u is then a solution to the Schrödinger equation $\Delta u = i(\partial u/\partial t)$. We set

$$S^* f(x) = \sup_{0<t<1} |S_t f(x)|, \qquad x \in \mathbb{R}^n.$$

We also introduce Sobolev spaces H_s by setting

$$H_s = \{ f \in \mathscr{S}'; \|f\|_{H_s} < \infty \}, \qquad s \in \mathbb{R},$$

where

$$\|f\|_{H_s} = \left(\int_{\mathbb{R}^n} (1 + |\xi|^2)^s |\hat{f}(\xi)|^2\, d\xi \right)^{1/2}.$$

We shall study the question for which values of s the estimate

$$\left(\int_B |S^* f(x)|^2\, dx \right)^{1/2} \leqslant C_B \|f\|_{H_s}, \ f \in \mathscr{S}(\mathbb{R}^n), \tag{1}$$

holds for all balls B in $\mathbb{R}^n$. We shall see that the inequality (1) has implications for the existence almost everywhere of $\lim_{t\to 0} u(x,t)$ for solutions u of the Schrödinger equation.

The inequality (1) and related questions have been studied by L. Carleson [2], B. E. J. Dahlberg and C. E. Kenig [4], C. E. Kenig and A. Ruiz [5], A. Carbery [1], M. Cowling [3], and E. M. Stein. They have proved that $s \geqslant n/4$ is a

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