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INFINITE DETERMINANTS, STABLE BUNDLES AND CURVATURE.

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Introduction. Let (X, ω) be a compact Kahler manifold of complex dimension n and E a holomorphic r-plane bundle over X. For simplicity we suppose $c_1(E) = 0$, although this restriction could easily be removed from our discussion. We say E is $[\omega]$-stable if every subsheaf $S \subset \mathcal{O}(E)$ with torsion-free quotient $S/\mathcal{O}(E)$ satisfies:

$$c_1(S) \cdot [\omega]^{n-1} < 0.$$

This condition clearly implies that E is indecomposable. A Hermitian metric h on E determines a unique compatible connection, with curvature F_h. Let

$$\hat{F}_h = F_h \cdot \omega \in \Omega_X^0(\text{End } E).$$

We say that h is *Hermitian-Einstein* or Hermitian Yang-Mills if $\hat{F}_h = 0$ everywhere. In 1980 Hitchin and Kobayashi suggested that the existence of such a metric should be related to the $[\omega]$-stability of E. They conjectured:

PROPOSITION 1. *If E is $[\omega]$-stable there exists a Hermitian Yang-Mills metric on E.*

(For the uniqueness and the converse see [3]). When X is a complex curve this is equivalent to a theorem of Narasimhan and Seshadri [8]. In [3] the author proved the proposition for bundles over complex algebraic surfaces. This paper extends the ideas in that reference in the light of various recent developments.

The most important development is the work of Uhlenbeck and Yau [10] who have proved Proposition 1 in full generality. Their proof is probably the most natural and displays clearly the role of stability. However it uses rather sophisticated analysis. In part III of this paper we extend the ideas of [3] to give an alternative proof of Proposition 1 for bundles over projective manifolds $X \subseteq \mathbb{CP}^N$ with a Hodge metric ω. By contrast this proof does not make explicit use of the definition of stability. It proceeds by induction on the dimension of X and uses the algebro-geometric work of Mehta and Ramanathan [7]. They prove:

PROPOSITION 2. *Suppose $[\omega]$ is the hyperplane class of $X \subseteq \mathbb{CP}^N$. A bundle $E \to X$ is $[\omega]$-stable if and only if there is $d_0 > 0$ such that for generic smooth*

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