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# ANOTHER PROOF OF A CONJECTURE OF S. P. NOVIKOV ON PERIODS OF ABELIAN INTEGRALS ON RIEMANN SURFACES

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**Introduction.** In this paper we present a proof of a well known conjecture due to S. P. Novikov. The conjecture states that given a complex symmetric matrix $\tau$, which is not in block form, and whose imaginary part is positive definite, then $\tau$ is the normalized period matrix for the abelian integrals of a compact connected Riemann surface if and only if the Riemann theta function $\theta(z, \tau)$ satisfies the so-called Kodomcev-Petviashvili equation, hereafter, K-P.

This conjecture has been recently proved by T. Shiota in [7]. The present note can be viewed as a shortcut to Shiota's proof. In our treatment we follow the spirit of Welters' paper [8] and of our previous paper [1]. We only use Riemann theta function, we never need the machinery of the abstract $\tau$-function and wave operators. As a consequence we do not have to check properties of quasiperiodicity of formal solution to the K-P hierarchy. Moreover as we already found in our previous work, we only need to consider a particularly nice sub-set of equations in the K-P hierarchy.

More importantly, our point of view is to exploit a little bit more the geometry in the given abelian variety $X$. In fact a source of simplification in our argument is to restrict the differential equations we are interested in, to the subscheme of $X$ defined by the vanishing of the theta function and of its $D_1$-derivative. Once this restriction is understood we can go back to Shiota's argument and see with clarity the path to follow. From Shiota's paper [7] we use the very skillful Lemma 7 which deals with the a priori possibility (which is, a posteriori, an impossibility) of the existence, inside the theta divisor, of integral curves for the $D_1$-flow.

It was in communicating with Gerald Welters that the ideas of §2 came to our minds. We thank him very much. We also thank Maurizio Cornalba for some useful hints concerning §3.

**§1. The geometrical interpretation of the K-P hierarchy.** In this section we shall introduce some notation and recall the basic results contained in [8] and [1].

Let $\mathscr{H}_g$ denote the Siegel upper-half phase of genus $g$. For $z \in \mathbb{C}^g$, $\tau \in \mathscr{H}_g$ and $n \in \frac{1}{2}\mathbb{Z}^g/\mathbb{Z}^g$ set

$$\theta\begin{bmatrix} n \\ 0 \end{bmatrix}(z, \tau) = \sum_{p \in \mathbb{Z}^g} \exp 2\pi i \left\{ \tfrac{1}{2}{}^t(p+n)\tau(p+n) + {}^t(p+n)z \right\}.$$

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