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A FUNCTIONAL EQUATION OF THE NON-ARCHIMEDIAN RANKIN CONVOLUTION

Dedicated to Yuri Ivanovich Manin on the occasion of his 50th birthday

A. A. PANCHISHKIN

§0. Introduction. Let p be a prime number and S a finite set of prime numbers containing p. The aim of this paper is to establish a functional equation satisfied by the S-adic L-functions, which are obtained by the non-Archimedian interpolation of the special values of the Rankin convolution of two elliptic cusp forms of different weight. Let N be an arbitrary positive integer. Let f be a cusp form of weight $k \geqslant 2$ for the congruence subgroup $\Gamma_0(N)$ with a character ψ modulo N, which is, in addition, a primitive form of conductor $C(f)$, i.e., normalized new form of the exact level $C(f)$ dividing N. Let g be another primitive form of conductor $C(g)$ and weight $l < k$ for $\Gamma_0(N)$ with a character ω. Write $e(z) = \exp(2\pi iz)$. Suppose that Fourier expansions of f and g are given by

$$f = \sum_{n=1}^{\infty} a(n)e(nz), \qquad g = \sum_{n=1}^{\infty} b(n)e(nz). \tag{0.1}$$

The Rankin convolution of f and g is denoted by

$$\mathcal{D}_N(s, f, g) = L_N(2s + 2 - k - l, \omega\psi)L(s, f, g), \tag{0.2}$$

where $L(s, f, g) = \sum_{n=1}^{\infty} a(n)b(n)n^{-s}$ and $L_N(2s + 2 - k - l, \omega\psi)$ is the Dirichlet L series of $\omega\psi$ with the Euler factors at the primes dividing N removed from its Euler product. It is known (from Rankin [14] and Selberg [16]) that $\mathcal{D}_N(s, f, g)$ has a holomorphic continuation over the whole complex plane and it satisfies a functional equation, which in the simplest case of $N = 1$ has the form

$$\Psi(s, f, g) = (-1)^k\Psi(k + l - 1 - s, f, g), \tag{0.3}$$

where

$$\Psi(s, f, g) = \gamma(s)\mathcal{D}_N(s, f, g) \tag{0.4}$$

with the Γ-factor $\gamma(s) = (2\pi)^{-2s}\Gamma(s)\Gamma(s + 1 - l)$.

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