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MODULAR FORMS IN CHARACTERISTIC ℓ AND SPECIAL VALUES OF THEIR L-FUNCTIONS

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In this paper we specialize our results in [A-S] to $G = GL_2$ to obtain information about congruences between classical holomorphic modular forms. In the final section we indicate how the methods can be adapted to prove congruences between special values of L-functions of modular forms of possibly different weights.

We begin with a study of the q-expansions in characteristic $\ell > 0$ of Hecke eigenforms of weight $k \geqslant 2$ and level N prime to ℓ. By a theorem of Eichler and Shimura this is equivalent to a study of the systems of eigenvalues of Hecke operators acting on the group cohomology of $\Gamma_1(N)$ (see §2). Using the functorial properties of cohomology, we show (Theorems 3.4 and 3.5) that the systems of Hecke eigenvalues (mod ℓ) occurring in the space, $\mathscr{M}_{>2}(\Gamma_1(N))$, of modular forms of level N and all weights > 2 coincide, up to twist, with those occurring in the space, $\mathscr{M}_2(\Gamma_1(N\ell))$, of weight two forms of level $N\ell$. In particular, we see that there are only finitely many systems of eigenvalues (mod ℓ) occurring in the infinite dimensional space $\mathscr{M}_{>2}(\Gamma_1(N))$, a fact proved by Jochnowitz [J] for prime $N \leqslant 17$.

Group cohomology has been used before in this theory. For example, a proof of Theorem 3.4(a) was given by Hida [H1] who refers to much earlier but unpublished work of Shimura [S1]. An account of Shimura's work can also be found in Ohta's article [O]. Kuga, Parry, and Sah [K-P-S] have proved similar statements and extended them to quaternionic groups. Haberland [Ha] has used cohomological methods in his study of congruences of Cartan type. Apparently new in our treatment is the use of the operator θ, a cohomological analog of "twisting" of modular forms, and of the operator Ψ (see the proof of 3.3(b)).

By taking the point of view of group cohomology we lose some structure. For example, we do not see the algebra structure of the space of modular forms. Nor do we see the Hodge decomposition of the cohomology groups. Notice also that we obtain no information about weight one forms. Nevertheless, the functoriality of group cohomology provides a powerful tool for studying congruences between eigenforms. Moreover, as many authors have noted [M, Mz, St], group cohomology is well suited for the study of p-adic properties of special values of L-functions.

In §4 we develop the theory of higher weight ($k > 2$) modular symbols, and examine the special values of L-functions attached to them. In Theorem 4.5 and

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