Vol. 52, No. 2 DUKE MATHEMATICAL JOURNAL © June 1985

ALGEBRAIC CYCLES AND VALUES OF L-FUNCTIONS II

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Recently A. Beilinson [1] and the author [3], [4] have independently formulated conjectures about relations between values of L-functions and algebraic cycles, analogous to the conjecture of Birch and Swinnerton-Dyer for points on abelian varieties. In their crude form, these conjectures equate the order of vanishing of the L-function associated to an odd dimensional cohomology group $H^{2r-1}(X_{\bar{k}}, \mathbf{Q}_l)$ of a smooth projective variety X over a number field k at the point $s = r$ to the rank of the group $A^r(X) = \operatorname{Ker}(CH^r(X) \otimes \mathbf{Q} \to H^{2r}(X_{\bar{k}}, \mathbf{Q}_l(r)))$. (Here $CH^r(X)$ is the Chow group of codimension r cycles on X defined over k modulo rational equivalence.) The reader may want to compare and contrast this with the Tate conjecture for cycles [19]. There is a more precise version of the conjecture involving a regulator term, but calculation of this regulator seems to require some new ideas and I shall have nothing more to say about it.

The purpose of this paper is threefold. First, I will formulate a refined crude conjecture relating the "coniveau filtration" on cohomology to a suitable filtration on the Chow group. This new conjecture has the virtue in many situations of highlighting certain "pieces" of the Chow group as especially interesting. In particular the nonvanishing of the Griffiths group of cycles homologous to 0 modulo algebraic equivalence can be "explained" arithmetically. My second objective is a "Coates–Wiles" type theorem applying to a limited range of situations where the L-function is associated to a Grössencharakter. Using work of Yager [20], I show that if there exists a cycle which is p-adically of infinite order in a suitable sense, then the L-function does vanish at the indicated point. This is reasonable philosophical evidence for the conjecture, but it is not much practical use since the infinite order condition is difficult to work with.

Finally I will discuss as example a variety found by C. Schoen [16]:

$$Y_0 : X_0^5 + \cdots + X_4^5 - 5X_0X_1X_2X_3X_4.$$

This hypersurface has 125 double points. Schoen shows that the resolved variety Y has $h^{2,1} = h^{1,2} = 0$, $h^{3,0} = h^{0,3} = 1$. Standard conjectures suggest that the L-function for H^3 is the Mellin transform of a modular form of weight 4 and

Received February 20, 1984. Partially supported by the National Science Foundation.