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THE DIFFUSION APPROXIMATION OF THE SPATIALLY HOMOGENEOUS BOLTZMANN EQUATION

TADAHISA FUNAKI

1. Introduction. In the present paper we study the diffusion approximation of the spatially homogeneous Boltzmann equation for the so-called repulsive inverse power law *soft potentials* without cutoff. This problem was first discussed by L. D. Landau [7] for Coulomb molecules to analyze the time evolution of a plasma (see also Lifshitz and Pitaevskii [9] and Spohn [10]). Motivated by his arguments, Maxwellian molecules were treated in a previous paper [2].

The methods employed here are probabilistic. We associate a jump process with the Boltzmann equation. It is then shown that under suitable scaling this process converges weakly to a stochastic process associated with a kind of nonlinear diffusion equation called the Landau equation.

The spatially homogeneous Boltzmann equation for soft potentials is given by

$$\frac{\partial u}{\partial t}(t,x)=\int_{(0,\pi)\times(0,2\pi)\times\mathbf{R}^3}\{u(t,x^*)u(t,y^*)-u(t,x)u(t,y)\}k(x,y)Q(d\theta)\,d\phi\,dy,$$
$$t\geqslant 0,\quad x=(x_i)_{i=1}^3\in\mathbf{R}^3,\quad (1.1)$$

where $x^*=x^*(x,y,\theta,\phi)$ and $y^*=y^*(x,y,\theta,\phi)$ are postcollision velocities of two molecules with precollision velocities x and y, respectively. The angles $\theta\in(0,\pi)$ and $\phi\in(0,2\pi)$ are, respectively, the colatitude and the longitude of x^* in a spherical coordinate system attached to a sphere in $\mathbf{R}^3$ with north pole x and south pole y. The Borel measure Q on $(0,\pi)$ satisfying $c(Q)=\int_0^\pi\theta^2Q(d\theta)<\infty$ and the nonnegative function $k(x,y)$ on $\mathbf{R}^3\times\mathbf{R}^3-\Delta$, $\Delta=\{(x,x)\in\mathbf{R}^6;\,x\in\mathbf{R}^3\}$, are determined from the dynamics of collision processes governed by the pair potential $U(\rho)$, $\rho=$ the distance between two molecules. In the case of repulsive inverse power law potentials:

$$U(\rho)=\text{const.}\,\rho^{-s+1}\qquad(s>3),$$

the function k is given by

$$k(x,y)=|x-y|^{(s-5)/(s-1)}.$$

When $3<s<5$, $s=5$ and $s>5$, the potentials are referred to, respectively, as

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