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ON FRENET FRAMES OF COMPLEX SUBMANIFOLDS IN COMPLEX PROJECTIVE SPACES

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Introduction. In his book *Meromorphic Functions and Analytic Curves* [10], H. Weyl remarked that Second Main Theorem, in the theory of holomorphic curves, is an unintegrated form of the classical Plücker formulas for projective algebraic curves. P. A. Griffiths points out, in an expository article [4], that these unintegrated formulas are of local nature and are essentially reflections of the Maurer–Cartan equations for the unitary group when applied to Frenet frames for holomorphic curves. With these two observations in mind, we try to extend the Frenet formalism to complex submanifolds in complex projective spaces.

We succeed in making such an extension by dealing with the curvature of a bundle $\Delta^{(k)*} \otimes \Delta^{(k+1)}$ over the submanifold (cf. §§4–5) induced by the k-th Gauss map. The bundle $\Delta^{(k)*} \otimes \Delta^{(k+1)}$ resembles the tangent bundle of a Grassmann manifold and has more complicated curvature. This is expressed in formula (5.8) which enables us to compute the Chern forms of various osculating metrics of the submanifold provided the dimension and codimension are not too high. An immediate consequence is a generalization (5.9) of Weyl's formula for holomorphic curves:

$$\gamma_1(\tilde{\Omega}^{(k)}) = -\nu_{k+1}\Phi_{k-1} + (\nu_k + \nu_{k+1})\Phi_k - \nu_k\Phi_{k+1}.$$

where $\tilde{\Omega}^{(k)}$ denotes the connection of the above mentioned bundle, Φ_k denotes the Kähler form of the kth osculating metric, and $\nu_k = \dim \Delta^{(k)}$.

As a simple application, we illustrate the theory for surfaces in P^5 with maximal second order osculating spaces (§6, Condition V). For this *special case* the following results are proved which give the flavor of the subject.

THEOREM 1 (Infinitesimal Plücker formulas). *Let ξ denote the pull back of the universal bundle of G_3^6 by the Gauss map, and $\Omega^{(k)}$ denote the connection of the k-th osculating metric. Then*

$$\gamma_1(\Omega^{(0)}) = 3\Phi_0 - \Phi_1,$$

$$\gamma_2(\Omega^{(0)}) = 3\Phi_0^2 - 2\Phi_0 \wedge \Phi_1 + \gamma_2(\xi),$$

$$3\gamma_1(\Omega^{(1)}) = -3\Phi_0 + 5\Phi_1,$$

$$11\gamma_2(\Omega^{(1)}) + \gamma_1^2(\Omega^{(1)}) = 6\Phi_0^2 - 14\Phi_0 \wedge \Phi_1 + 11\Phi_1^2 + \gamma_2(\xi).$$

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