

SUPERCUSPIDAL REPRESENTATIONS OF SL_n OVER A p -ADIC FIELD: THE TAME CASE

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§1. Introduction. The determination of the discrete series of irreducible unitary representations of a reductive group G defined over a p -adic field F is one of the major unsolved problems in the representation theory of p -adic groups. A complete classification has been given in only a few special cases (listed below). In contrast to the discrete series for real reductive Lie groups, the discrete series of representations of p -adic groups splits into two classes which must be treated by radically different techniques.

(1) Supercuspidal representations: irreducible unitary representations whose matrix coefficients are compactly supported (mod the center of G);

(2) Generalized special representations: irreducible unitary representations whose matrix coefficients are square integrable (mod the center of G), and which are subrepresentations of representations induced from a proper parabolic subgroup of G .

Further complications arise from the relationship between the residual characteristic p of F and the rank of G .

The cases in which the discrete series have been classified are the following (more or less in chronological order):

$SL_2(F)$, p odd ([GG], [Sk]);

$PGL_2(F)$, p odd ([Si]);

$GL_2(F)$, p odd ([JL]);

$GL_2(F)$, $p = 2$ ([K], [K'], [H']);

$GL_l(F)$, l prime, $\text{char } F = 0$ ([C]);

$GL_n(F)$, $(n, p) = 1$, $\text{char } F = 0$ ([H], [BZ], [Z], [M]);

$SL_2(F)$, $p = 2$ ([KS']).

For more details on the known results about the construction of the discrete series for p -adic groups see [KS], [Sa]. It is of interest to note that all the supercuspidal representations which have been constructed for p -adic groups may be induced from open, compact (mod center) subgroups of G .

In this paper, we study the supercuspidal representations of $SL_n(F)$ in the case when $(n, p) = 1$ and $\text{char } F = 0$ (the tame case). The ultimate goal is the construction of all supercuspidal representations of $SL_n(F)$ as representations induced from explicitly given representations of compact open subgroups. We do this in "almost all" cases; the exceptions reducing to questions about the representation theory of GL_n and SL_n over finite fields.

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