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INEQUALITIES FOR PEANO MAXIMAL FUNCTIONS AND MARCINKIEWICZ INTEGRALS

SAGUN CHANILLO AND RICHARD L. WHEEDEN

§1. Introduction. The purpose of this paper is to derive relations between the various maximal functions which arise in the distribution function estimates for Marcinkiewicz integrals given in [6], and to use these relations to simplify some of the global norm inequalities given there. A summary of the results in [6] as well as those of this paper is given in [7].

We begin by listing definitions of the pertinent maximal functions. Let $\mathsf{R}^{n+1}_+ = \{(x,y): x \in \mathsf{R}^n, 0 < y < \infty\}$ be the $(n+1)$-dimensional upper Euclidean half-space and let $F(x,y)$ be harmonic in R^{n+1}_+. If $0 < q < \infty$ and $0 < \alpha \leqslant 2$, we consider the Peano maximal function

$$\mathscr{P}^*_{\alpha,q}(F)(x) = \sup_{s,h>0} \begin{cases} h^{-\alpha}\left(h^{-n}\displaystyle\int_{|u|<h} |F(x+u,s) - F(x,s)|^q\,du\right)^{1/q}, & 0 < \alpha \leqslant 1 \\ h^{-\alpha}\left(h^{-n}\displaystyle\int_{|u|<h} |F(x+u,s) - F(x,s) \right. \\ \qquad\qquad \left. - u \cdot \nabla_1 F(x,s)|^q\,du\right)^{1/q}, & 1 < \alpha \leqslant 2. \end{cases}$$

Here ∇_1 stands for the gradient in the x variables, $\nabla_1 = \langle \partial/\partial x_1, \dots, \partial/\partial x_n \rangle$, as distinguished from the full gradient $\nabla = \langle \nabla_1, \partial/\partial y \rangle$, and "$\cdot$" denotes the usual Euclidean dot product. We have made the restrictions $0 < \alpha \leqslant 2$ and $0 < q < \infty$ for simplicity; the definition for other values of α and for $q = \infty$ is given in §2. $\mathscr{P}^*_{\alpha,q}(F)$ is a "lifted" version of the standard Peano maximal function $\mathscr{P}_{\alpha,q}(F)$ defined for a function $F(x)$ on R^n as follows:

$$\mathscr{P}_{\alpha,q}(F)(x) = \sup_{h>0} \begin{cases} h^{-\alpha}\left(h^{-n}\displaystyle\int_{|u|<h} |F(x+u) - F(x)|^q\,du\right)^{1/q}, & 0 < \alpha \leqslant 1 \\ h^{-\alpha}\left(h^{-n}\displaystyle\int_{|u|<h} |F(x+u) \right. \\ \qquad\qquad \left. - F(x) - u \cdot A(x)|^q\,du\right)^{1/q}, & 1 < \alpha \leqslant 2, \end{cases}$$

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