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ON THE PERIOD MAP FOR SURFACES WITH $K_S^2 = 2, p_g = 1, q = 0$ AND TORSION $\mathbf{Z}/2\mathbf{Z}$

P. OLIVERIO

Summary. In questo lavoro diamo un controesempio al teorema di Torelli per superficie S con $K_S^2 = 2, p_g = 1, q = 0$ e torsione $\mathbf{Z}/2\mathbf{Z}$. Se $\mu : H^1(S, T_S) \to H^1(S, \Omega_S^1)$ é l'applicazione infinitesimale di Torelli, allora si ha dim(ker μ) $\geqslant 1$ e infatti 1, genericamente. In tal modo, se $\Psi : \mathscr{M} \to \Gamma \backslash D$ é l'applicazione dei periodi, la fibra generale ha dimensione 1.

Introduction. In this paper we study the period map [5] of surfaces with $K^2 = 2$, $p_g = 1$, $q = 0$ and torsion $\mathbf{Z}/2\mathbf{Z}$. Surfaces S with $K_S^2 = 2$, $p_g = 1$, $q = 0$ and torsion $\mathbf{Z}/2\mathbf{Z}$ have been described in [4]. One obtains such surfaces as quotient of weighted complete intersections of type $(4, 4)$ in $\mathbf{P}(1, 1, 1, 2, 2)$ by the involution $\tau : (W, X_1, X_2, Z_3, Z_4) \to (W, -X_1, -X_2, -Z_3, -Z_4)$. Further in [4] it is proved that their moduli space $\mathscr{M}$ is irreducible and rational of dimension 16. Using the fact that when K_S is ample the Kuranishi family is smooth and can be explicitly described, we compute the differential μ of the period map with methods similar to the ones used in [2]. It turns out that dim(ker μ) $\geqslant 1$ and indeed $= 1$ on Zariski open subset of $\mathscr{M}$. Therefore, if we denote by $\Gamma \backslash D$ the classifying space for pure Hodge structure of weight 2 and Hodge numbers $(1, 18, 1)$, $\Gamma \backslash D$ has dimension 18 and the period map $\Psi : \mathscr{M} \to \Gamma \backslash D$ has a general fibre of dimension 1.

Notation. $W \in H^\circ(S, \mathscr{O}_S(K_S))$ is the unique (up to constants) nonzero section.

R is the graded ring $\mathbf{C}[W, X_1, X_2, Z_3, Z_4]$, where $\deg W = 1$, $\deg X_1 = 1$, $\deg X_2 = 1$, $\deg Z_3 = 2$, $\deg Z_4 = 2$.

$R(Y) = \bigoplus_{\mathscr{M}=0}^{\infty} H^\circ(Y, \mathscr{O}(mK_Y))$ is the canonical ring of a smooth projective variety Y.

$h^i(Y, L) = \dim H^i(Y, L)$, where L is a coherent sheaf on a projective variety Y.

$\mathbf{P} = \mathbf{P}(1, 1, 1, 2, 2) = \mathrm{Proj}(R)$.

$S_n = \{$homogeneous polynomials of degree n in $\mathbf{C}[X_1, X_2]\}$.

S is a smooth projective surface over $\mathbf{C}$ with $K_S^2 = 2$, $p_g(S) = 1$, $q(S) = 0$ and torsion $\mathbf{Z}/2\mathbf{Z}$.

§1. Let B be the base of the Kuranishi family of deformations of S. The tangent space to B at the point 0 corresponding to the surface S is identified in a

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