

NONLINEAR MICROLOCAL ANALYSIS OF SEMILINEAR HYPERBOLIC SYSTEMS IN ONE SPACE DIMENSION

JEFFREY RAUCH AND MICHAEL REED

TABLE OF CONTENTS

INTRODUCTION	397
CHAPTER 1. HARMONIC ANALYSIS OF $(H^s)^k$ SPACES	
§1.1 Basic properties	411
§1.2 Multiplicative properties	415
§1.3 Local and microlocal spaces	427
§1.4 Invariance under nonlinear maps	432
CHAPTER 2. REGULARITY OF SOLUTIONS OF SEMILINEAR HYPERBOLIC EQUATIONS	
§2.1 Regularity valid everywhere	443
§2.2 The basic propagation theorem	445
§2.3 The general regularity theorem: formulation and examples	448
§2.4 The general regularity theorem: proof	456
§2.5 Resolving the solid cones	464
REFERENCES	475

Introduction. This paper is devoted to the study of the singularities of solutions to semilinear strictly hyperbolic systems in one space variable, that is, systems of the form

$$A_0(x, t) \frac{\partial u}{\partial t} + A_1(x, t) \frac{\partial u}{\partial x} = f(x, t, u)$$

where $(x, t) \in \mathbb{R} \times \bar{\mathbb{R}}_+$, $u(x, t) \in \mathbb{C}^m$, the A_i are smooth $m \times m$ matrix-valued functions, and $f: \mathbb{R} \times \bar{\mathbb{R}}_+ \times \mathbb{C}^m \rightarrow \mathbb{C}^m$ is smooth. If the system is strictly hyperbolic, then $\det A_0 \neq 0$ and $\det(\tau A_0 + \xi A_1)$ has m distinct real roots for all (x, t) and $\xi \neq 0$. By a smooth change of dependent variables, the system can be recast in the form

$$\left(\frac{\partial}{\partial t} + \lambda_j(x, t) \frac{\partial}{\partial x} \right) u_j = f_j(x, t, u) \quad j = 1, \dots, m \quad (1)$$

Received January 20, 1982. Research of first author partially supported by NSF Grant #MCS-79-01857. Research of second author partially supported by NSF Grant #MCS-78-02179.