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ON SELF-LINKED CURVES

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Let V be a subvariety of $\mathbf{P}^n$ of codimension two. We wish to study the geometric situation where twice V is the intersection, X, of two hypersurfaces, i.e., where there exist two hypersurfaces which meet exactly on V and which make contact with each other with multiplicity two, along V. We will study the phenomena algebraically using the techniques of liaison of Peskine–Szpiro [P-S]. We will hence assume that V is a locally Cohen–Macaulay subscheme of $\mathbf{P}^n$ of codimension two, which most of the time will be generically a local complete intersection. V will be called self-linked or linked to itself if we can find a complete intersection X containing V such that V is residual to itself in X, i.e., $\mathcal{I}_V = \mathrm{Ann}(\mathcal{I}_V/\mathcal{I}_X)$, where $\mathcal{I}_V$ is the ideal sheaf of V. The simplest nontrivial example of self-linkage is the case of the twisted cubic curve in $\mathbf{P}^3$, which, even though not a complete intersection, is set-theoretically a complete intersection by means of a quadric and a cubic which touch along C (see Remark 2, below). In [C], Catanese presents a method of Gallarati where, starting from this example, more complicated examples are created by preserving self-linkage under liaison. This method, thus, creates many examples of self-linked curves which are projectively Cohen–Macaulay. The other commonly known example of self-linkage is the example of two Kummer surfaces touching along a smooth curve of degree 8 and genus 5. This classical example has been recently treated by Barth [B-2] in a study of the Mumford–Horrocks bundle on $\mathbf{P}^4$, and it is a curve that is not projectively normal. Gallarati's inductive procedure applies again to give new examples generated through liaison from this example. One question that arises immediately is to find those liaison equivalence classes of curves in $\mathbf{P}^3$ which contain a self-linked curve. Now a curve Y in $\mathbf{P}^3$ has an associated module over the ring $S = k[X_0, X_1, X_2, X_3]$ defined as $M(Y) = \bigoplus_\nu H^1(\mathbf{P}^3, \mathcal{I}_Y(\nu))$ which is a liaison "invariant" in the sense that if Y is linked to Y', then $M(Y')$ is the dual $\mathrm{Ext}^4_S(M(Y), S)$ of $M(Y)$ up to shifts in grading [R-1]. Hence an immediate necessary condition for a curve Y to be self-linked is that its liaison invariant $M(Y)$ be self-dual up to grading. Since the liaison invariant characterises liaison classes [R-1], this is a restrictive condition on the liaison class for it to contain a self-linked curve. As an example, the above curve of genus 5 and degree 8 lies in the liaison class of the module $k \oplus k$.

We use the method of preserving self-linkage by liaison (see Proposition 6) to reduce the complexity of the self-linked curve being studied (Proposition 8). In the first example of Gallarati, this would amount to going from a general

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