Vol. 48, No. 4 DUKE MATHEMATICAL JOURNAL © December 1981

THE BOUNDARY BEHAVIOR OF AUTOMORPHIC FORMS

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1. Introduction. We consider a Fuchsian group Γ acting in the upper half plane $H = \{z : \operatorname{Im} z > 0\}$ and with the following properties.

Γ is finitely generated and of the first kind. (1.1)

Γ contains a cyclic subgroup fixing ∞ generated by the translation $z : \rightarrow z + \lambda$. (1.2)

Every parabolic element in Γ is conjugate, in Γ, to a translation. (1.3)

We denote by D the Ford fundamental region for Γ which is contained in the strip $\{z : |\operatorname{Re} z| \leqslant \lambda/2\}$, [3, p. 77]. For positive integer q we consider meromorphic automorphic forms on Γ of weight q (dimension $-2q$). Thus for such a form we have

$$f(V(z))(V'(z))^q = f(z) \tag{1.4}$$

for all $z \in H$ and all $V \in \Gamma$. Such a form is required to have a limit as z goes to infinity in D. If this limit is zero we note that, for $\operatorname{Im} z$ large enough,

$$f(z) = \sum_{m=\mu}^{\infty} a_m \exp[2\pi \operatorname{im} z\lambda^{-1}] \tag{1.5}$$

where μ is a positive integer and we say that f has a zero of order μ at infinity.

In this paper we consider the behavior of an automorphic form on approach within a stolz angle to a real number. This question has been studied extensively in certain special cases. In [2] Cohn considers certain forms for the modular group on approach to real numbers whose continued fraction expansions have bounded partial denominators and some related results were obtained by Epstein and Lehner (see [6] p. 333). Somewhat later Rosen [8] made similar investigations for the Hecke groups of the first kind in which he used an arithmetic characterization (an analog of the continued fraction expansion) for the limit points of the group. Quite recently Sheingorn [9] has obtained much more complete information in the case of the modular group.

Received April 21, 1981. Revision received July 2, 1981.