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UNSTABLE ORBITAL INTEGRALS ON SL(3)

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Introduction. Let G be a connected reductive group over a global field F. In [4] Langlands associates to G a family of connected reductive groups H over F and suggests how to use these groups H in the study of harmonic analysis on $G(F)\backslash G(\mathbf{A}_F)$. (The global suggestions are in R. Langlands, Les débuts d'une formule des traces stables, Lectures at ENSJF, in preparation.) Langlands also introduces the groups H for a connected reductive group G over a local field F and suggests how to use them in the study of harmonic analysis on $G(F)$. This theory has been worked out in two cases:

(1) $G = \mathrm{SL}(2)$ (and certain related groups), F local or global (see [3]),

(2) G arbitrary, F real or complex (see [6, 7, 8, 9]).

One aspect of Shelstad's theory for real groups is the matching of functions f on $G(\mathbf{R})$ and f' on $H(\mathbf{R})$ so that certain linear combinations of orbital integrals of f are equal to certain linear combinations of orbital integrals of f'. For this matching we need to assume that the embedding of L-groups ${}^LH \to {}^LG$ of Proposition 1 in [4] exists (this will be the case if the center of ${}^LG^0$ is connected), and in fact the precise form of the matching depends on the choice of embedding. Using the groups H to study harmonic analysis for groups over global fields will require the matching of functions by orbital integrals for non-archimedean local fields F as well, and if G, H are unramified (that is, quasi-split over F and split over an unramified extension of F), then the matching of spherical functions should be given by the homomorphism of Hecke algebras dual to ${}^LH \to {}^LG$. The purpose of this paper is to verify a precise form of the last statement for one particular case.

We take F to be a non-archimedean local field, L an unramified cubic extension of F, $G = \mathrm{SL}(3)$, $H = \ker(\mathrm{Res}_{L/F}\mathbf{G}_m \xrightarrow{\text{norm}} \mathbf{G}_m)$. We have $H(F) = \{x \in L^\times : N_{L/F}x = 1\}$. Let W_F be the Weil group of F. The L-group ${}^LG$ of G is $W_F \times \mathrm{PGL}_3(\mathbf{C})$. The L-group ${}^LH$ of H is $W_F \ltimes S$ where S is the quotient of $\mathbf{C}^\times \times \mathbf{C}^\times \times \mathbf{C}^\times$ by $\mathbf{C}^\times$ embedded diagonally. The group W_F acts on S through the quotient group $\mathrm{Gal}(L/F)$ by cyclic permutations of the three factors of $\mathbf{C}^\times$. There is an obvious embedding of S in $\mathrm{PGL}_3(\mathbf{C})$ obtained by mapping (z_1, z_2, z_3) into the diagonal matrix with entries z_1, z_2, z_3, and this embedding can be extended to a unique embedding ${}^LH \to {}^LG$ such that the restriction of ${}^LH \to {}^LG$ to W_F is $w \mapsto w \times s_w$ where s_w is the identity matrix if w maps to the identity in

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