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THE ARITHMETIC OF LUBIN-TATE DIVISION TOWERS

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I. Introduction. In this paper we continue the study of Lubin-Tate division towers initiated in $[C_2]$. We will be principally concerned with the Iwasawa-Wiles homomorphism ψ from coherent sequences of units to coherent sequences of elements in the dual of the image of the Lubin-Tate logarithm. Our notation will be the same as in $[C_2]$.

Thus K is a local field of any characteristic, H a finite unramified extension of K, π a uniformizing parameter of K and $\mathfrak{T}$ a Lubin-Tate formal group associated to π and K. Then H_n is the field obtained by adjoining the π^{n+1} division points $\mathfrak{T}_n$ of $\mathfrak{T}$ to H. First we will recall the definition of ψ. Recall (see [W] or [L]) that $(\ ,\ )_n$ is a pairing

$$(\ ,\ )_n : \mathfrak{T}(\mathfrak{p}_n) \times H_n^* \to \mathfrak{T}_n$$

where $\mathfrak{T}(\mathfrak{p}_n)$ is the group that $\mathfrak{T}$ associates with $\mathfrak{p}_n$ the maximal ideal in the maximal order of H_n. This pairing is defined as follows: Let $a \in \mathfrak{p}_n$, $b \in H_n^\times$ and let $\alpha \in \Omega$ such that $[\pi^{n+1}](\alpha) = a$. (Ω is the completion of a fixed algebraic closure of K.) Then $H_n(\alpha)$ is an abelian extension of H_n and if we let σ_b denote the image of b in $\mathrm{Gal}(H_n(\alpha)/H_n)$ via the Artin map then

$$(a,b)_n = \sigma_b(\alpha)[-]\alpha$$

where $[-]$ denotes subtraction in $\mathfrak{T}$. Now let λ be the logarithm of $\mathfrak{T}$, v a generator of $T_{\mathfrak{T}}$ the Tate module of $\mathfrak{T}$ and $T_{n/K}$ the trace from H_n to K. Let

$$\mathfrak{X}_n = \{\beta \in H_n : T_{n/K}(\lambda(\alpha)\beta) \in \mathfrak{O} \quad \text{for all} \quad \alpha \in \mathfrak{p}_n\}$$

where $\mathfrak{O}$ is the ring of integers of K. Let $X_n' = N_{2n+1,n}(H_{2n+1}^\times)$. Then Proposition 7 of [W] asserts that there exists a unique homomorphism

$$\psi_{v,n} : X_n' \to \mathfrak{X}_n / \pi^{n+1}\mathfrak{X}_n$$

such that

$$(a,b)_n = [T_{n/K}(\lambda(a), \psi_{v,n}(b))](v_n)$$

for all a in Y_n and b in X_n' where v_n denotes the nth component of v. Moreover,

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